EXERCISE 16.2

Question 1:
A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Solution:
When a die is rolled, the sample space is given by

$$S = \{1,2,3,4,5,6\}$$

Accordingly, $E = \{4\}$ and $F = \{2,4,6\}$

It is observed that $E \cap F = \{4\} \neq \phi$

Therefore, E and F are not mutually exclusive events.

Question 2:
A die is thrown. Describe the following events:
(i) A: a number less than 7
(ii) B: a number greater than 7
(iii) C: a multiple of 3
(iv) D: a number less than 4
(v) E: an even number greater than 4
(vi) F: a number not less than 3
Also find $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, E \cap F', F'$

Solution:

When a die is thrown, the sample space is given by $S = \{1,2,3,4,5,6\}$
Accordingly:
(i) A: a number less than 7

$$A = \{1,2,3,4,5,6\}$$

(ii) B: a number greater than 7

$$B = \phi$$

(iii) C: a multiple of 3

$$C = \{3,6\}$$

(iv) D: a number less than 4

$$D = \{1,2,3\}$$

(v) E: an even number greater than 4

$$E = \{6\}$$

(vi) F: a number not less than 3

$$F = \{3,4,5,6\}$$

$A \cup B = \{1,2,3,4,5,6\}$
$A \cap B = \phi$
$B \cup C = \{3,6\}$
$E \cap F = \{6\}$
$D \cap E = \phi$
$A - C = \{1,2,4,5\}$
$D - E = \{1,2,3\}$
$F' = \{1,2\}$
$E \cap F' = \phi$

Question 3:

An experiment involves rolling a pair of dice and recording the number that comes up. Describe the following events.
A: the sum is greater than 8
B: 2 occurs on either die
C: The sum is at least 7 and multiple of 3.
Which pairs of these events are mutually exclusive?

Solution:

When a pair of dice is rolled, the sample space is given by

$$S = \{(x,y) : x, y = 1,2,3,4,5,6\}$$

It is observed that

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Hence,

$$A = \{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$$
$$B = \{(1,2),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$$
$$C = \{(3,6),(4,5),(5,4),(6,3),(6,6)\}$$

It can be observed that

$A \cap B = \phi$

$B \cap C = \phi$

$C \cap A = \{(3,6),(4,5),(5,4),(6,3),(6,6)\} \neq \phi$

Hence events A and B and events B and C are mutually exclusive.

Question 4:

Three coins are tossed once. Let A denote the event "three heads show", B denote the event "two heads and one tail show". C denote the event "three tails show" and D denote the event 'a head shows on the first coin". Which events are

(i) mutually exclusive?
(ii) simple?
(iii) compound?

Solution:

When three coins are tossed, the sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Accordingly,

$A = \{HHH\}$

$B = \{HHT,\ HTH,\ THH\}$

$C = \{TTT\}$

$D = \{HHH,\ HHT,\ HTH, HTT\}$

We can observe that

$A \cap B = \phi$

$A \cap C = \phi$

$A \cap D = \{HHH\} \neq \phi$

$B \cap C = \phi$

$B \cap D = \{HHT, HTH\} \neq \phi$

$C \cap D = \phi$

(i) Events A and B; events A and C; events B and C; and events C and D are all mutually exclusive.
(ii) If an event has only one sample point of a sample space, it is called a simple event. Thus, A and C are simple events.

(iii) If an event has more than one sample point of a sample space, it is called a compound event. Thus, B and D are compound events.

Question 5:

Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii) Two events, which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Solution:

When three coins are tossed, the sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

(i) Two events that are mutually exclusive can be
A: getting no heads and
B: getting no tails

This is because sets $A = \{TTT\}$ and $B = \{HHH\}$ are disjoint.

(ii) Three events that are mutually exclusive and exhaustive can be
A: getting no heads
B: getting exactly one head
C: getting at least two heads
i.e.,

$A = \{TTT\}$

$B = \{HTT, THT, TTH\}$

$C = \{HHH, HHT, HTH, THH\}$

This is because $A \cap B = B \cap C = C \cap A = \phi$ and $A \cup B \cup C = S$

(iii) Two events that are not mutually exclusive can be
A: getting three heads
B: getting at least 2 heads
i.e.,

$A = \{HHH\}$

$B = \{HHH, HHT, HTH, THH\}$

This is because $A \cap B = \{HHH\} \neq \phi$

(iv) Two events which are mutually exclusive but not exhaustive can be
A: getting exactly one head
B: getting exactly one tail
i.e.,

$A = \{HTT, THT, TTH\}$

$B = \{HHT, HTH, THH\}$

This is because $A \cap B = \phi$ but $A \cup B \neq S$

(v) Three events that are mutually exclusive but not exhaustive can be
A: getting exactly three heads
B: getting one head and two tails
C: getting one tail and two heads
i.e.,

$A = \{HHH\}$

$B = \{HTT, THT, TTH\}$

$C = \{HHT, HTH, THH\}$

This is because $A \cap B = B \cap C = C \cap A = \phi$ but $A \cup B \cup C \neq S$

Question 6:

Two dice are thrown. The events A, B and C are as follows:
A: getting an even number on the first die.
B: getting an odd number on the first die.
C: getting the sum of the numbers on the dice ≤ 5
Describe the events

(i) A' (ii) not B (iii) A or B
(iv) A and B (v) A but not C (vi) B or C
(vii) B and C (viii) $A \cap B' \cap C'$

Solution:

When two dice are thrown, the sample space is given by

$$S = \{(x, y) : x, y = 1, 2, 3, 4, 5, 6\}$$

Hence,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Accordingly,

$$A=\begin{Bmatrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

$$B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}$$

$$C=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(2,1),\\(2,2),(2,3),(3,1),(3,2),(4,1)\end{Bmatrix}$$

(i) $$A'=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}=B$$

(ii) $$\text{not } B=B'=\begin{Bmatrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}=A$$

(iii) $$A \text{ or } B=A\cup B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}=S$$

(iv) A and B $=A\cap B=\phi$

(v) A but not C $$=A-C=\begin{Bmatrix}(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),\\(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

(vi) $$B \text{ or } C=B\cup C=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),\\(2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)(4,1),\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}$$

(vii) B and C $=B\cap C=\{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii) $$C' = \begin{Bmatrix} (1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4), \\ (3,5),(3,6),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1), \\ (6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Therefore,

$$A \cap B' \cap C' = A \cap A \cap C' = A \cap C' = \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Question 7:

Refer to question 6 above

Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

State true or false: (give reason for your answer)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii) $A = B'$

(iv) A and C are mutually exclusive

(v) A and B' are mutually exclusive

(vi) A', B', C are mutually exclusive and exhaustive.

Solution:

When two dice are thrown, the sample space is given by

$$S=\{(x,y):x,y=1,2,3,4,5,6\}$$

Hence,

$$S=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

Accordingly,

$$A=\begin{Bmatrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

$$B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}$$

$$C=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(2,1),\\(2,2),(2,3),(3,1),(3,2),(4,1)\end{Bmatrix}$$

(i) It is observed that $A\cap B=\phi$
Therefore, A and B are mutually exclusive.
Thus, the given statement is true.

(ii) It is observed that $A\cap B=\phi$ and $A\cup B=S$
Therefore, A and B are mutually exclusive and exhaustive.
Thus, the given statement is true.

(iii) It is observed that

$$B'=\begin{Bmatrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}=A$$

Thus, the given statement is true.

(iv) It is observed that $A\cap C=\{(2,1),(2,2),(2,3),(4,1)\}=\phi$

Therefore, A and C are not mutually exclusive.
Thus, the given statement is false.

(v) It is observed that $A \cap B' = A \cap A = A$

Therefore, $A \cap B' \neq \phi$; A and B' are not mutually exclusive.
Thus, the given statement is false.

(vi) It is observed that $A' \cup B' \cup C = S$
However,

$B' \cap C = \{(2,1),(2,2),(2,3),(4,1)\} \neq \phi$

Therefore, events A', B', and C are not mutually exclusive and exhaustive.
Thus, the given statement is false.