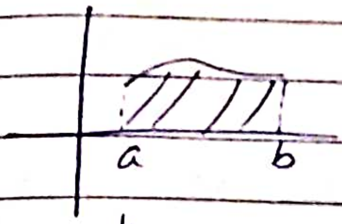
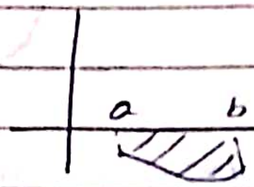
AOI

Area under the curve

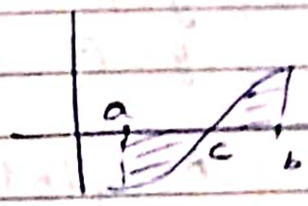
Area bounded by curve $= \int_a^b |f(x)|\,dx$



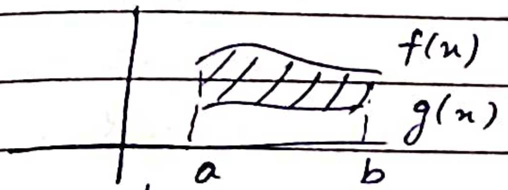
$A = \int_a^b f(x)\,dx$



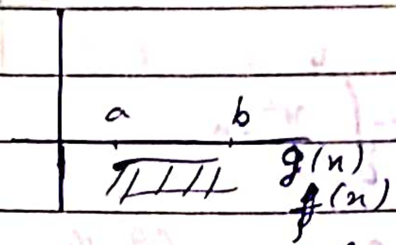
$A = \left| \int_a^b f(x)\,dx \right|$



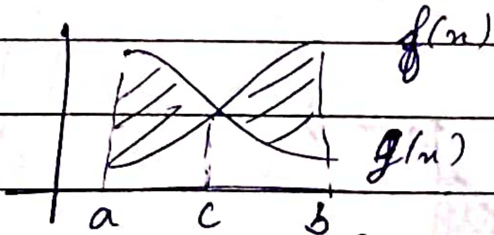
$A = \int_a^c -f(x)\,dx + \int_c^b f(x)\,dx$



$A = \int_a^b f(x) - g(x)\,dx$



$A = \int_a^b g(x) - f(x)\,dx$



$$A = \int_a^c g(x) - f(x)\,dx + \int_c^b f(x) - g(x)\,dx$$

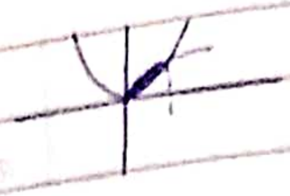
$A = \int y\,dx \qquad A_{y\,axis} = \int x\,dy$

~~Feel~~

Ques Find area bounded by

1) $y = x^2$

$y = \sqrt{x}$



$A = \int_0^1 \sqrt{x}\,dx - \int_0^1 x^2\,dx$

$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

2) $y^2 = 4x$

$x^2 = 4y$

$A = \int_0^4 \sqrt{4x}\,dx - \int_0^4 \frac{x^2}{4}\,dx$

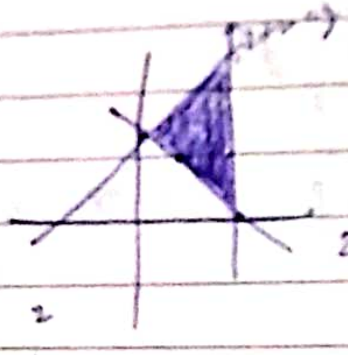
$= \sqrt{4} \times \frac{2}{3} \times 8 - \frac{64}{12} \frac{16}{3}$

$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$

3) $y = x+2$

$y = 2-x$

$x = 2$



$A = \int_0^2 (x+2)\,dx - \int_0^2 (2-x)\,dx$

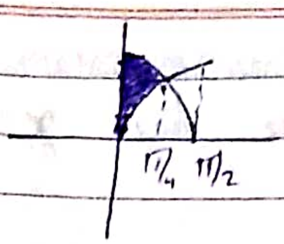
$2 + 2 = 4$

4) $y = \sin x$

$y = \cos x$

Find area bounded by these 2 curves with y-axis in 1st quad. $(x \in 0, \frac{\pi}{2})$

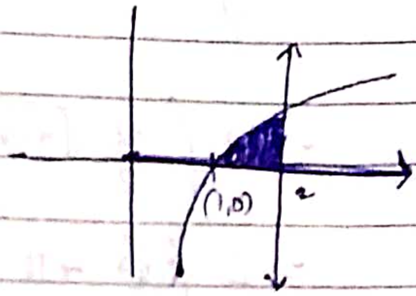
$$A = \int_0^{\pi/4} \cos x - \int_0^{\pi/4} \sin x$$



$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

Ques Find area bounded by $y = \log x$, $y = 0$ and $x = 2$

$$A = \int_1^2 \log x$$



$$= x \log x - x$$

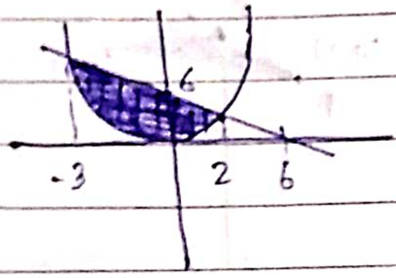
$$= 2 \log 2 - 2 + 1$$

$$= 2 \log 2 - 1$$

$$= \log \frac{4}{e}$$

Ques Find area bounded by $y = x^2$ and $y = 6 - x$

$$A = \int_{-3}^{2} 6 - x - \int_{-3}^{2} x^2$$



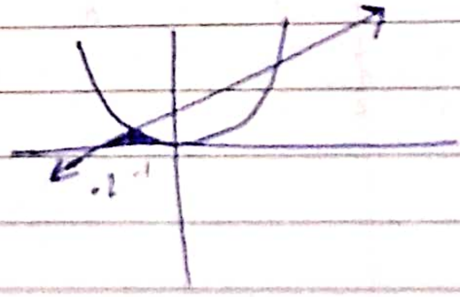
$$30 - \frac{1}{2} \times 13 - \frac{35}{3}$$

$$30 - \frac{13}{2} - \frac{35}{3}$$

$$\frac{180 - 39 - 70}{6} = \frac{71}{6}$$

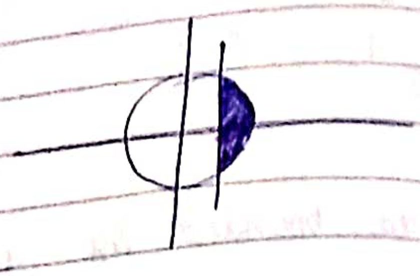
Ques Find area bounded by $y = x^2$, $y = x + 2$ and x-axis

$$A = \int_{-2}^{-1} x + 2 \, dx + \int_{-1}^{0} x^2 \, dx$$



$$2 - \frac{3}{2} + \frac{1}{3} = \frac{5}{6}$$

Ques Find area of smaller portion of circle $x^2+y^2=4$ cut off by the line $x=1$



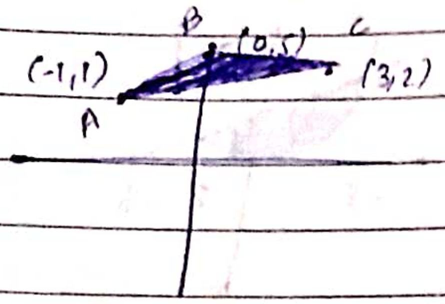
$$A = 2\int_1^2 \sqrt{4-x^2}\,dx$$

$$= 2\left(\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right)_1^2$$

$$= 2\left(-\frac{\sqrt{3}}{2} + \pi - \frac{\pi}{3}\right)$$

$$= 2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

Find area of Δ formed by $A(-1,1)$ $B(0,5)$ $C(3,2)$



AB : $\frac{y-5}{x} = 4$

$$\boxed{y = 4x+5}$$

BC : $\frac{y-5}{x} = -1$

$$\boxed{y + x = 5}$$

CA : $\frac{y-1}{x+1} = \frac{1}{4}$

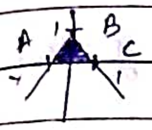
$4y - 4 = x+1$

$$\boxed{y = \frac{x+5}{4}}$$

$$A = \int_{-1}^{0} AB + \int_{0}^{3} BC - \int_{-1}^{3} AC$$

$$\int_{-1}^{0} 4x+5 + \int_{0}^{3} 5-x - \frac{1}{4}\int_{-1}^{3} x+5$$

Ques Find area of the region bounded by x axis and $|x| + y = 1$



AB : $y - 1 = x$

$\boxed{y = x + 1}$

BC : $y - 1 = (-1)x$

$\boxed{y = 1 - x}$

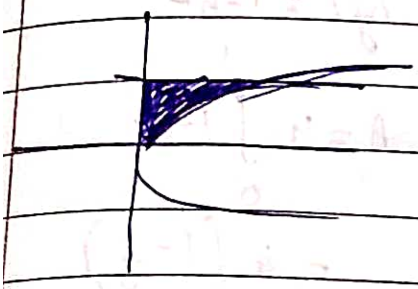
OR we can directly open mode:

$$A = 2\int_0^1 1 - |x|$$

$$= 2\int_0^1 (1-x)\,dx$$

$$= 2\left(1 - \left(\frac{1}{2}\right)\right) = 1$$

Find area bounded by $y^2 = 4ax$, $y = 2a$ and y-axis

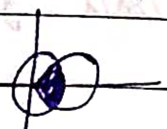


$$A = \int_0^{2a} \frac{y^2}{4a}\,dy$$

$$= \frac{1}{4a} \times \frac{1}{3}\, 8a^3 = \frac{2}{3}a^2$$

$x^2 + y^2 = 1$ $\quad (x-1)^2 + y^2 = 1$

Find area of region bounded by above curves.



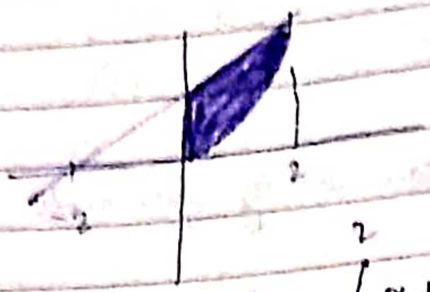
$$A = 2\int_0^{1/2} \sqrt{1-(x-1)^2} + \int_{1/2}^{1} \sqrt{1-x^2}\,dx$$

$$= 2\left[\left(\frac{x-1}{2}\right)\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right] + \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]$$

$$= 2\left(\frac{1}{2}\times\frac{\pi}{2} - \frac{1}{4}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{\pi}{6}\right) + 2\left(\frac{-1}{4}\times\frac{\sqrt{3}}{2} - \frac{\pi}{12} + \frac{\pi}{4}\right)$$

$$= 4\left(-\frac{\sqrt{3}}{8} - \frac{\pi}{12} + \frac{\pi}{4}\right)$$

Ques Find area of region bounded by $x-y+2=0$, $x=\sqrt{y}$ and y-axis

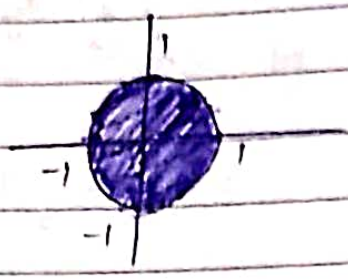


$$A = \int_0^2 x+2 - \int_0^2 x^2$$

$$= 2+4-\frac{8}{3}$$

$$= \frac{10}{3}$$

Ques Find area of region enclosed in $|y| = 1-x^2$

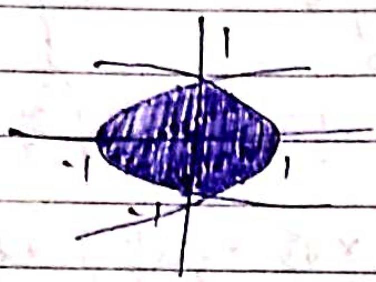


$$A = 4\int_0^1 1-x^2$$

$$= 4\left(1-\frac{1}{3}\right)$$

$$= 8/3$$

Ques Find area enclosed b/w $y^2=x+1$ and $y^2=1-x$



$$A = 4\int_0^1 \sqrt{x+1}$$

$$4\times\frac{2}{3}\left((2)^{3/2}-1\right)$$

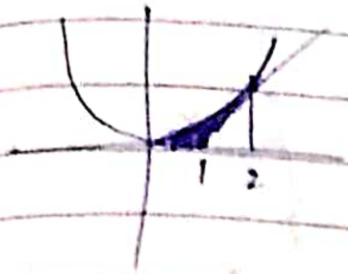
$$=\frac{8}{3}\left[(2)^{3/2}-1\right]$$

Ques Find area bounded by $y = x^2$, x-axis, tangent to $y = x^2$ at $x = 2$

$\frac{dy}{dx} = 2x$

$m = 4$

$y = 4$



$y - 4 = 4(x-2)$

$\boxed{y = 4x - 4}$

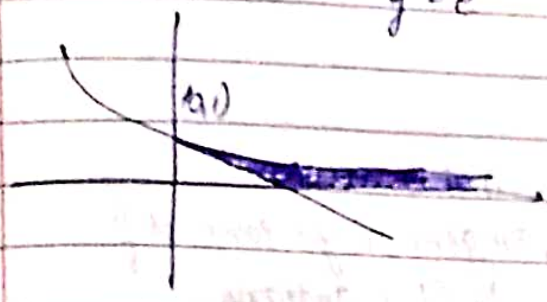
$A = \int_0^2 x^2\,dx - \int_1^2 4x - 4$

$\frac{8}{3} + 4\left[\frac{3}{2} - 1\right]$

~~$\frac{8}{3} - 6 + 4$~~ $\quad \frac{8}{3} + 6 = \frac{26}{3}$

Ques Find area of region bounded by

$y = e^{-x}$, $y = 1 - x$ and x-axis

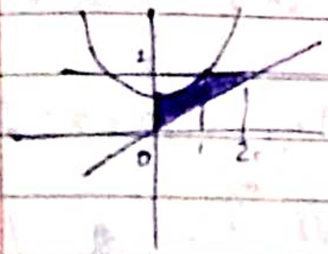


$A = \int_0^\infty e^{-x}\,dx - \int_0^1 1 - x\,dx$

$\left(\frac{e^{-x}}{-1}\right)_0^\infty - \left(x - \frac{x^2}{2}\right)_0^1$

$\frac{1}{2}$

Ques $y = x^2 + 1$, $y = x$, $x = 0$, $y = 2$. Find Area

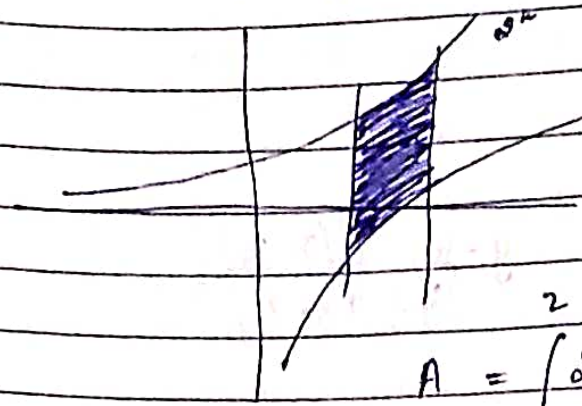


$A = \int_0^2 y\,dy - \int_1^2 \sqrt{y-1}$

$2 - \frac{2}{3} = \frac{4}{3}$

Date

Q. Find area of region bounded by $x=2$, $x=\frac{1}{2}$, $y=2^x$, $y=\ln x$



$$A = \int_{1/2}^{2} 2^x - \int_{1/2}^{2} \ln x$$

$$= \left(\frac{2^x}{\ln 2}\right)_{1/2}^{2} - \left(x\ln x - x\right)_{1/2}^{2}$$

$$= \left(\frac{4}{\ln 2} - 2\ln 2 + 2 - \frac{\sqrt{2}}{\ln 2} + \frac{1}{2}\ln 2 - \frac{1}{2}\right)$$

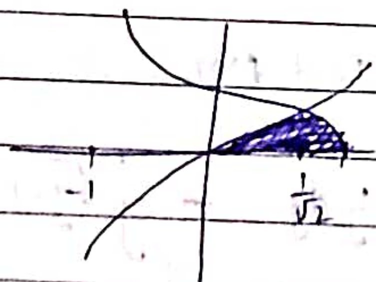
$$= \left(\frac{4-\sqrt{2}}{\ln 2}\right) - \frac{5}{2}\ln 2 + \frac{3}{2}$$

HW

Find area enclosed by

1. $y = \sin^{-1}x$, $y = \cos^{-1}x$ and x-axis
2. $y = \tan x$, x-axis, tangent to $y = \tan x$ at $\frac{\pi}{4}$
3. $y = \ln(x+e)$, $x = \ln\left(\frac{1}{y}\right)$ and x axis
4. $y = |x+3|$ with x-axis from $x=-6$ to $x=0$

(1)



$$A = \int_{0}^{1/\sqrt{2}} \sin^{-1}x + \int_{1/\sqrt{2}}^{1} \cos^{-1}x$$

$$\left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^{1/\sqrt{2}} + \left[x\cos^{-1}x - \sqrt{1-x^2}\right]$$

$$\frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}$$

$A \Rightarrow \sqrt{2} - 1$

② $y = \tan x$

$\frac{dy}{dx} = \sec^2 x$

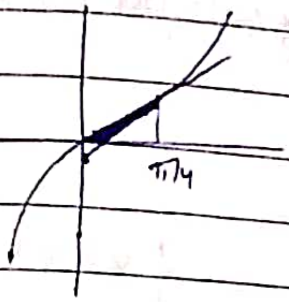
$m = 2$

$x = \frac{\pi}{4}$

$y = 1$

$(y-1) = 2\left(x - \frac{\pi}{4}\right)$

$y = 2x - \frac{\pi}{2} + 1$



$$A = \int_0^{\pi/4} \tan x - \int_{\pi/4 - \frac{1}{2}}^{\pi/4} 2x - \frac{\pi}{2} + 1$$

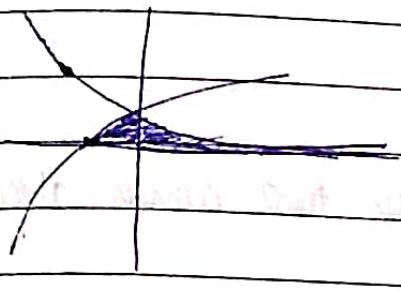
$$\ln\sqrt{2} - \left[\left(\frac{\pi}{4}\right)^2 - \left(\frac{\pi}{4} - \frac{1}{2}\right)^2 + \left(1 - \frac{\pi}{2}\right)\frac{1}{2}\right]$$

$$\ln\sqrt{2} - \left[\frac{1}{4} + \frac{\pi}{4} + \frac{1}{2} - \frac{\pi}{4}\right]$$

$$\ln\sqrt{2} - \frac{1}{4}$$

③ $y = \ln(x+e)$ x-axis $x = \ln\left(\frac{1}{y}\right)$

$e^x = \frac{1}{y}$ $y = e^{-x}$

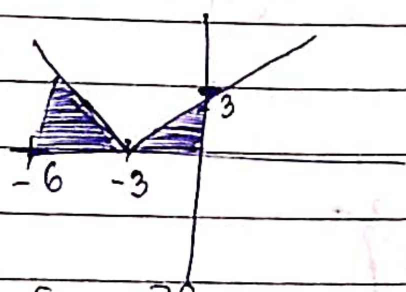


$$A = \int_{1-e}^{0} \ln(x+e) + \int_0^{\infty} e^{-x}$$

$$\left[(x+e)\ln(x+e) - (x+e)\right]_{1-e}^{0} + \left(\frac{e^{-x}}{-1}\right)_0^{\infty}$$

$1 + 1 = 2$

④ $y = |x+3|$



$$A = 2\int_{-3}^{0} x + 3 \, dx$$

$$2\left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

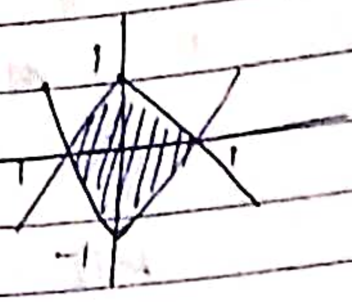
$$2\left[\frac{-9}{2} + 9\right] = 9$$

Ques $y = \frac{3}{|x|}$ $y = 2 - |2-x|$

Find area bounded b/w these 2 curves

Ques Find area bounded by $y = |x| - 1$

$y = 1 - |x|$



$\frac{1}{2} \times 2 \times 1 \times 2$

$= 2$

$$A = 2\int_{-1}^{0}(1+x)\,dx + 2\int_{0}^{1}(1-x)\,dx$$

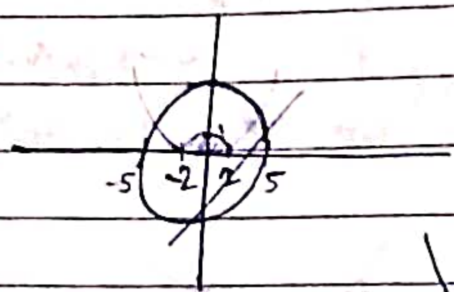
$$2\left[~~\frac{1}{2}~~ + 1 - \frac{1}{2}\right]$$

$= 2$ (ANS)

Ques $x^2 + y^2 = 25$

$4y = |4 - x^2|$

Find area bounded b/w these two curves above x-axis

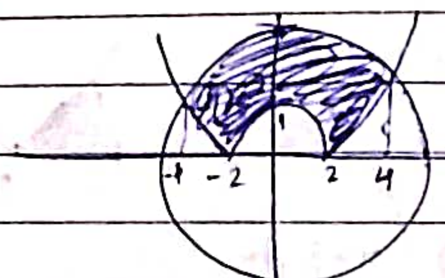


$1 - \frac{x^2}{4}$

$\left(\frac{4-x^2}{4}\right)^2 = 25 - x^2$

$16 + x^4 - 8x^2 = 400$

$-16x^2$



$16 + x^4 + 8x^2 = 400$

~~$z^2 + 8z - 384 = 0$~~

$(x^2+4)^2 = 400$

$x = \pm 4$

~~$\frac{25\pi}{4}$~~ $\int_{0}^{2}$

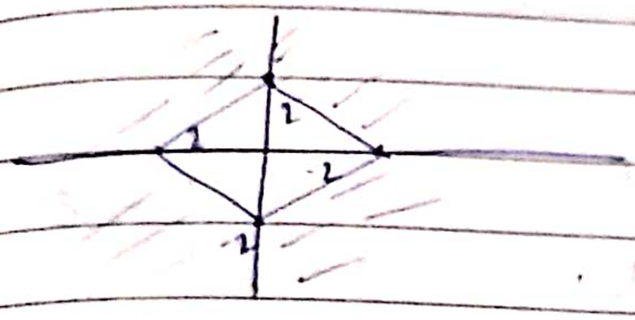
$$2\int_0^4 \sqrt{25-x^2} - 2\int_0^4 \frac{|4-x^2|}{4}\,dx$$

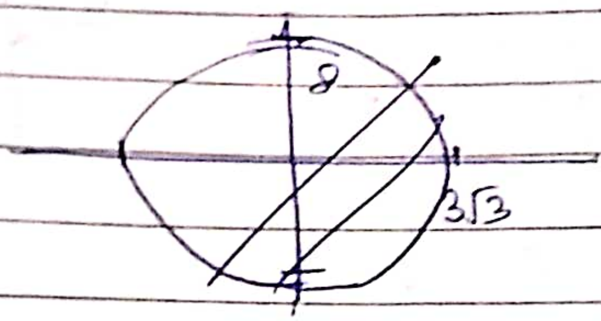
$$\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^4$$

$$2\times 3 + \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right) - \frac{25}{2}\sin^{-1}(0)$$

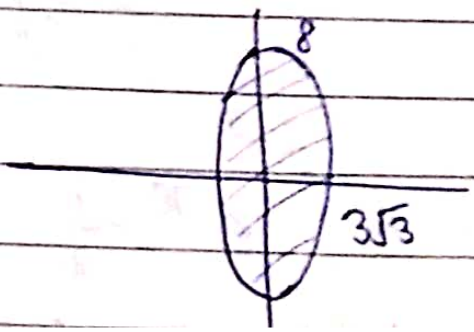
ans $|x| + |y| \geq 2$

$$\frac{x^2}{27} + \frac{y^2}{64} \leq 1$$





$$24\sqrt{3}\,\pi - (2\sqrt{2})^2$$



→ Check domain and range

→ Find points where the curve crosses x and y axis

→ If all powers of x are even then graph is symmetrical about y axis. If [Or terms has |x|]

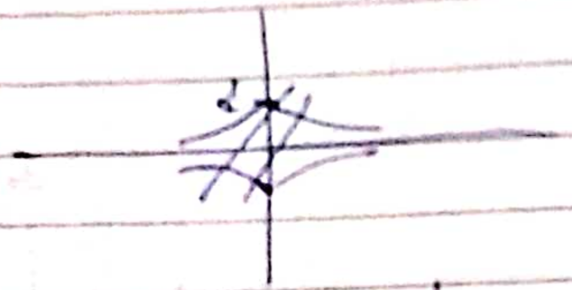
→ If all powers of y are even then graph is symmetrical about x-axis [Or terms has |y|]

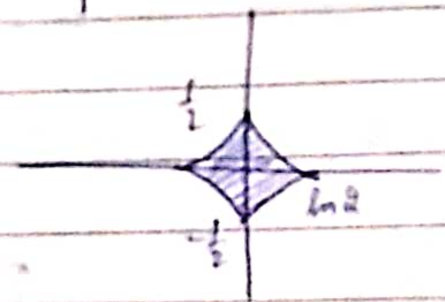
→ Find interval of ↑s or ↓s and maxima-minima

→ If eqn of curve remains same by interchanging x and y then the curve is symmetrical about y = x

Ques Find area of region represented by

$$|y| + \frac{1}{2} \le e^{-|x|}$$





$$\frac{1}{2} = e^{-x}$$

$$\ln 2 = x$$

$$A = 4\int_0^{\ln 2} e^{-x} - \frac{1}{2}$$

$$4\left[-\left[e^{-x}\right]_0^{\ln 2} - \frac{\ln 2}{2}\right]$$

$$4\left(+\frac{1}{2} - \frac{\ln 2}{2}\right)$$

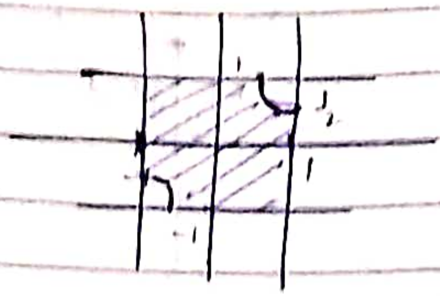
$$+2 - 2\ln 2$$

Ques Find area of region represented by

$|x| \le 1$

$|y| \le 1$

$xy \le \frac{1}{2}$



~~$2+2$~~

$$\int_{\frac{1}{2}}^{1} \frac{1}{2x}\, dx$$

~~$\frac{1}{2} \ln 2$~~

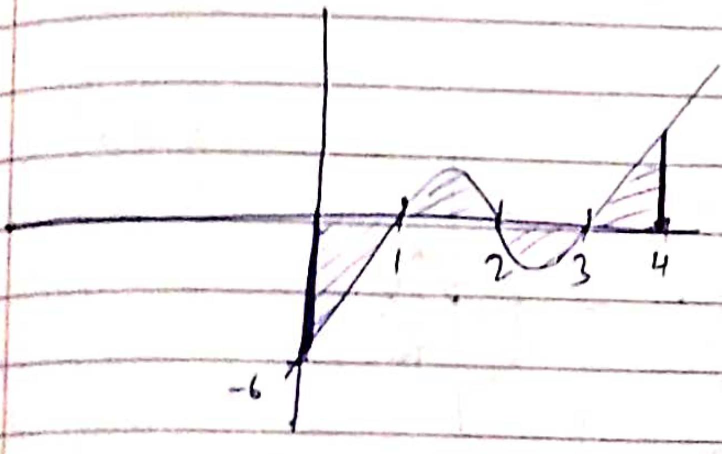
~~$2 + 2\left(2 - \frac{1}{2}\ln 2\right)$~~

~~$6 - \ln 2$~~

$2 + 1 + 2\left(\frac{1}{2}\ln 2\right)$

$\underline{3 + \ln 2}$

Ques



$y = (x-1)(x-2)(x-3)$

$(x^2 + 2 - 3x)(x-3)$

$x^3 - 3x^2 + 2x - 3x^2 - 6 + 9x$

$x^3 - 6x^2 - 7x - 6$

$$A = \int_0^1 x^3 - 6x^2 - 7x - 6$$

$\left(\frac{1}{4} - \frac{6}{3} - \frac{7}{2} - 6\right)$

$-8 - \frac{7}{2} + \frac{1}{4}$

$\frac{-32 - 14 + 1}{4} = \frac{-45}{4}$

$\frac{16}{4} - \frac{6 \times 8}{3} - \frac{7 \times 4}{2} - 6(2)$

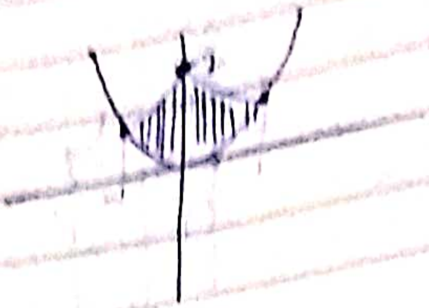
$4 - 16 - 14 - 12$

$-8 - 30$

-38

Ques $y = x^2$

$y = \frac{2}{1+x^2}$



$y = \frac{2}{1+x^2}$

$-(2x)/2$

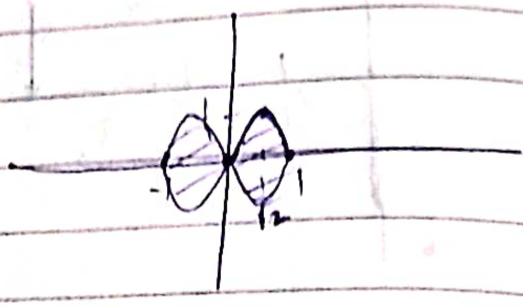
$-1/x$

$A = 2\int_0^1 \frac{2}{1+x^2}\,dx - \int_0^1 x^2\,dx$

$2\left[2 \times \frac{\pi}{4} - \frac{1}{3}\right]$

$\pi - 2/3$ (ANS)

Ques $y^2 + x^4 = x^2$



$y^2 = x^2 - x^4$

$y = \sqrt{x^2 - x^4}$

$y' = \frac{1}{2\sqrt{x^2 - x^4}}\; 2x - 4x^3$

$x - 2x^3$

$x(1 - 2x^2)$

$\frac{1}{\sqrt{2}} \quad \frac{1}{2} - \frac{1}{4}$

$\left[0, \frac{1}{\sqrt{2}}\right]$

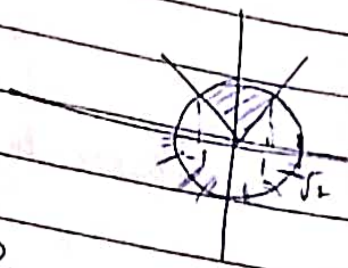
$A = 4\int_0^1 \sqrt{x^2 - x^4}$

$= 2\int_0^1 2x\sqrt{1-x^2}\,dx$

$= 2\int_0$

Ques $y = \sqrt{2-x^2}$

$y = |x|$



$y^2 + x^2 = 2$

$2x^2 = 2$

$$A = 2\left[\int_0^1 \sqrt{2-x^2} - \int_0^1 \frac{1}{2}\times 1\times 1\right]$$

$$A = 2\left[\frac{x}{2}\sqrt{2-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|_0^1 - \frac{1}{2}\right]$$

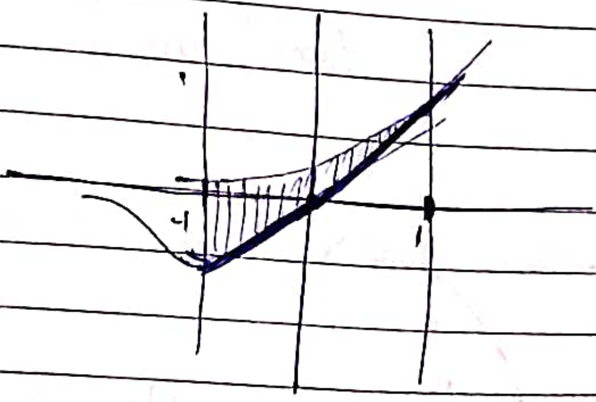
$$A = 2\left[\frac{1}{2} + \frac{\pi}{4} - \frac{1}{2}\right]$$

$$A = \frac{\pi}{2}$$

Ques $y = xe^x$

$y = e^x$

$x = \pm 1$



xe^x

$xe^x + e^x$

$e^x(1+x)$

$xe^x + e^x, e^x$

$xe^x + 2e^x$

$e^x(x+2)$

$$A = \int_{-1}^{1} (e^x - xe^x)\,dx$$

$e - \frac{1}{e}$ $\quad xe^x$

$-xe^x + \int e^x$

$-e - \frac{1}{e}$ $\quad 2e + \frac{2}{e} - e + \frac{1}{e}$

$e + \frac{1}{e}$

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Ques If Area bounded by

$y = f(x)$ $\quad f(x) > 0$

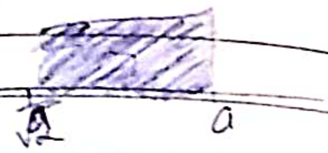
$x = \frac{1}{\sqrt{2}}$

$x = a$ is given by $\sec^2 a - \frac{1}{2}$

Also $a \in \left(\frac{1}{\sqrt{2}}, \frac{\pi}{2}\right)$

find $f(x)$

$$\int_{\frac{1}{\sqrt{2}}}^{a} f(x)\,dx = \sec^2 a - \frac{1}{2}$$



$f(a) = 2\sec^2 a \tan a$

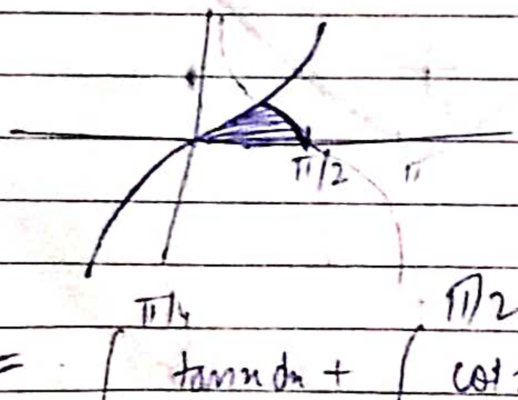
$f(x) = 2\sec^2 x \tan x$

Ques Find area enclosed by

$y = \tan x$

$y = \cot x$

from $\left(0, \frac{\pi}{2}\right)$



$$A = \int_0^{\pi/4} \tan x\,dx + \int_{\pi/4}^{\pi/2} \cot x\,dx$$

$\log|\sec x| + \log|\sin x|$

$\log\sqrt{2} + \log\sqrt{2}$

$2\log\sqrt{2} = \log 2$

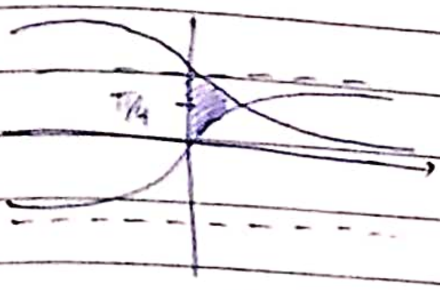
Ques Find area enclosed by

$y = \tan^{-1}x$

$y = \cot^{-1}x$

y = axis

$y = 0$, $y = \pi/2$



$$A = \int_0^{\pi/4} \tan y \, dy + \int_{\pi/4}^{\pi/2} \cot y \, dy$$

$$A = \log 2$$

* The area enclosed by $y = f(x)$, x-axis from $x = a$ to $x = b$ = Area enclosed by $y = f^{-1}(x)$, y-axis, $y = a$ to $y = b$.

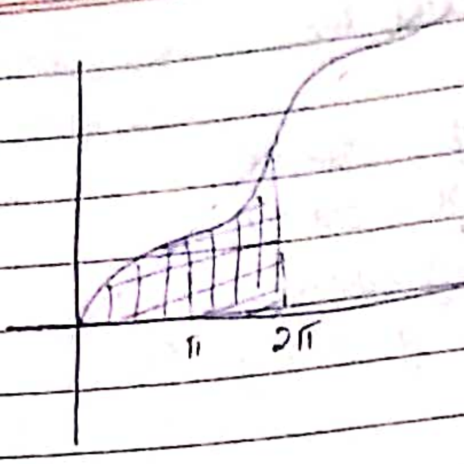
Ques $f(x) = x + \sin x$

$g(x) = f^{-1}(x)$

Find area bounded by $g(x)$ with y-axis from $y = 0$ to $y = 2\pi$

$$A = \int_0^{2\pi} x + \sin x \, dx$$

$$= 2\pi^2 + 4$$



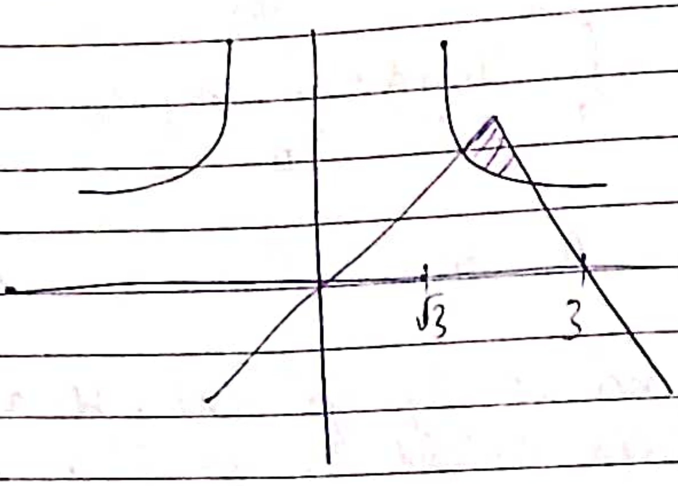
$y = x + \sin x$

A=

Ques

$y = \frac{3}{|x|}$

$y = 2 - |2-x|$



$y = x$

$y = \frac{3}{x}$

$$A = \int_{\sqrt{3}}^{2} x - \frac{3}{x} + \int_{2}^{\sqrt{3}} 4 - x - \frac{3}{x}$$

$$\frac{1}{2} - 3\log\left|\frac{2}{\sqrt{3}}\right| + 4(\sqrt{3}-2) + \frac{1}{2} - 3\log\left|\frac{\sqrt{3}}{2}\right|$$

$1 + 4\sqrt{3} - 8$

$4\sqrt{3} - 7$

$$\frac{1}{2} - 3\log\left(\frac{2}{\sqrt{3}}\right) + 4(\sqrt{3}-3) - 3 + 3\log|\sqrt{3}$$

$$\frac{1}{2} - 3\log\left|\frac{2}{\sqrt{3}}\right| + 4 - \frac{5}{2} - 3\log\left(\frac{3}{2}\right)$$

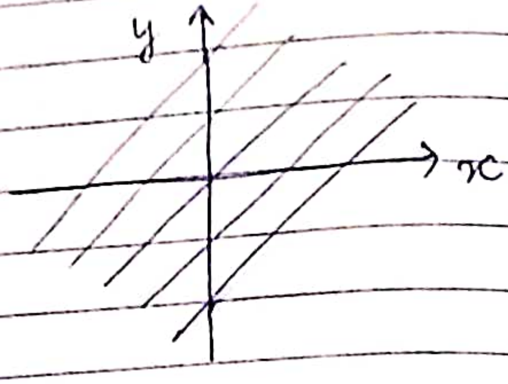
$$\Rightarrow 2 - \frac{3}{2}\log 3$$

01/08/19

Differential Equation

Independent variable x
Dependent variable y
diff. coff. of y wrt x $\Rightarrow \frac{dy}{dx} \Big/ \frac{d^2y}{dx^2} \Big/ \frac{d^3y}{dx^3}$

rep. family of curve with slope 3 $\left\{ \frac{dy}{dx} = 3 \right.$



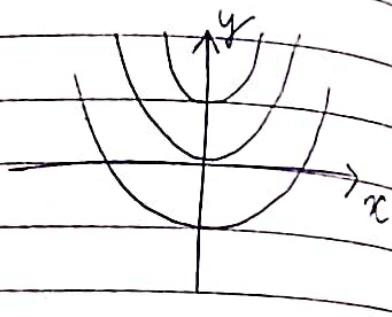
$$\int dy = \int 3dx$$

$$y = 3x + c$$

$$\frac{dy}{dx} = x$$

$$\int dy = \int x\,dx$$

rep. family of parabola with axis -y-axis $\Big\}$ $y = \frac{x^2}{2} + c$



* **Order and degree of Diff. equation**

Order :- It is the order of highest order derivative present in the equation. It is +ve integer.

Degree :- It is exponent of highest differential coff. when the diff. equation is written as polynomial in all diff. coefficients.

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for eg: $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 6 = 0$

Order: 3
Degree: 1

$x\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^4 + x^2 = 0$

Order: 3
Degree: 2

$\frac{d^2y}{dx^2} = \frac{dy}{dx} \ln x$

order: 2
degree: 1

$\frac{d^2y}{dx^2} = \ln\left(\frac{dy}{dx}\right)$

Order: 2
Degree: not defined

$\left(\frac{d^2y}{dx^2}\right) = \sin\left(\frac{dy}{dx}\right)$

Order: 2
Degree: not defined

$\ln\left(\frac{dy}{dx}\right) = x+y$

$\frac{dy}{dx} = e^{x+y}$

Order: 1
Degree: 1

Q.

$$\left(\frac{d^2y}{dx^2}\right) + \sin\left(\frac{dy}{dx}\right) = 0$$

Order : 2

Degree : Not defined

$$\left(\frac{d^2y}{dx^2}\right) = \frac{dy}{dx} + 3$$

Order : 2

Degree : 2

$$y = \frac{dy}{dx} + \frac{3}{dy/dx}$$

$$\left(\frac{dy}{dx}\right) y = \left(\frac{dy}{dx}\right)^2 + 3$$

order : 1

degree : 2

$$\left[1 + (y')^2\right]^{3/2} = 3y''$$

$$\left(1 + (y')^2\right)^3 = (3y'')^2$$

Order : 2

degree : 2

$\frac{dy}{d}$

$$y = 2y' + \sqrt{a^2(y')^2 + b^2}$$

$$(y - 2y')^2 = a^2(y')^2 + b^2$$

$$y^2 + 4y'^2 - 4yy' = a^2y'^2 + b^2$$

$(a \neq 2)$

$$y'^2(a^2 - 4) + b^2 + 4yy' - y^2 = 0$$

Degree : 2

Order : 1

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$\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$

order : 1

degree : 1

Linear and non-linear differential equation :

First write the diff. eqⁿ as polynomial in diff. coeff. If all the diff. coeff. and (y) dependent variable are in first power and there is no term having product of dependent variable and diff. coeff., then it is linear diff. eqⁿ.

eg:
$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2 = \cos x$$

→ Order : 2

→ degree : 1

→ Linear diff. eqⁿ

$$\frac{d^2y}{dx^2} = y\frac{dy}{dx} + x$$

→ Order : 2

→ degree : 1

→ Non linear

$$\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} + y^2 = 2$$

→ Order : 2

→ degree : 2

→ Non linear

$$\frac{d^6x}{dy^6} + \ln\left(\frac{d^2y}{dx^2}\right) = 0$$

→ order : 6

→ degree : Not defined

→ Non linear

$$y\frac{d^2x}{dy^2} = y^2 + 1$$

→ order : 2

→ degree : 1

→ ~~Non~~ linear

$$y + \frac{dy}{dx} = \frac{1}{6}\int y\,dx$$

$$\frac{dy}{dx} + \frac{d^2y}{dx^2} = \frac{1}{6}y$$

→ order : 2

→ degree : 1

→ linear

* Formation of Differential Eq^n

– Form a diff. eq^n which represents family of lines with y intercept 6

~~$y = \frac{dy}{dx}x + 6$~~

$$y = mx + 6$$

$$\frac{dy}{dx} = m$$

$$\frac{dy}{dx} = \frac{y-6}{x}$$

$$\boxed{y = x\frac{dy}{dx} + 6}$$

* We eliminate the arbitrary constant.

⇒ No. of essential arbitrary constants decides order of diff eqⁿ

* Form diff. eqⁿ representing following family of curves:

1. Family of ∨ having vertex at origin and axis along +ve y axis.

$$y^2 = 4ax$$

$$2y\frac{dy}{dx} = 4a$$

$$2y\frac{dy}{dx} = \frac{y}{x}$$

$$y = x\frac{dy}{dx}$$

$$x^2 = 4ay$$

$$\frac{x^2}{4a} = y$$

$$\frac{dy}{dx} = \frac{2x}{4a}$$

$$\frac{dy}{dx} = \frac{2xy}{x^2}$$

$$\boxed{y = \frac{x}{2}\frac{dy}{dx}}$$

2. Family of hyperbola having centre at origin and focii on x-axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{b^2x}{a^2y}$$

$$\frac{d^2y}{dx^2} = \frac{a^2b^2y - a^2b^2x\frac{dy}{dx}}{a^4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{a^2b^2y - \frac{dy}{dx}a^2b^2x}{a^4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{b^2(y - x\,dy/dx)}{a^2y^2}$$

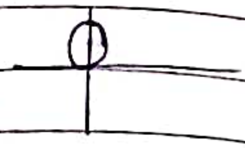
$\frac{dy}{dx} = \frac{b^2}{a^2}$ ...

$$\frac{d^2y}{dx^2} = \frac{1}{y^2}\frac{dy}{dx}\frac{x\,y}{x}\,(y - x\,dy/dx)$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{x}\frac{dy}{dx} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2}$$

3. Family of circles touching x-axis at origin and having centre on y-axis.

~~$x^2 + (y^2 = 1$~~



$$x^2 + (y-K)^2 = K^2$$

$$x^2 + y^2 = 2yK$$

$$2x + 2yy' = 2y'K$$

$$x + yy' = y'K$$

$$1 + y''y + (y_1)^2 = y''K$$

$$1 + y''(y - K) + (y')^2 = K$$ ×

$$x + yy' = y'K$$

$$x = y'(K - y)$$

$x^2 + \left(y + \left(\frac{x}{y}\right)\right)^2 =$ ~~$x^2 = \frac{y}{2}($~~

$$x^2 + y^2 = 2y\left(\frac{x + yy_1}{y'}\right)$$

$$\boxed{x^2y' + y^2y' = 2xy + 2y^2y'}$$

4. Family of curves $y = a\cos(x+b)$ where a, b are arbitrary constants

$$y = a\cos(x+b)$$
$$y' = -a\sin(x+b)$$
$$y'' = -a\cos(x+b)$$
$$y'' = -y$$
$$\boxed{y'' + y = 0}$$

5. $y = a\sin(bx+c)$

a, b, c - arb const

$$y = a\sin(bx+c)$$
$$y' = ab\cos(bx+c)$$
$$y'' = -ab^2(\sin bx+c)$$
$$y''' = -ab^3(\cos bx+c)$$
$$y''' = -y'b^2$$
$$y''' = +y' \frac{y'''}{y'}$$
$$y'y'' = yy''$$

6. $y = ae^{bx}$

a, b - const

$$y' = abe^{bx}$$
$$y'' = ab^2e^{bx}$$
$$y'' = \frac{(y')^2}{y}$$

7. $y = c(x-c)^2$

c - const

$$y = x^2c + c^3 - 2xc^2$$
$$y' = 2xc - 2c^2$$
$$y' = 2c(x-c)$$

$$\frac{y}{y'} = \frac{(x-c)}{2}$$
$$c = x - \frac{2y}{y'}$$

$$y' = \left(2x - \frac{4y}{y'}\right)\left(x - \frac{2y}{y'}\right)$$
$$(y')^3 = 2xy' - 8y^2$$

Ques $y = ae^{x} + be$ ...
Find order of diff. eqⁿ representing this family of curve.

order = 3

Ques Find order of diff. eqⁿ representing following family of curves:

1) $y = a\cos x + b\sin x \cos c + b\cos x +$

$\cos x\,(a+b) + (b\cos c)\sin x$

$P\cos x + Q\sin x$

order = 2

2) $y = (K_1 + K_2)\tan(x + K_3) - K_4 e^{x} + K_5$

order = 4

3) $y = C_1 x + (C_2 + C_3)e^{\ln x} + C_4 \cos(x + C_5)$

order = 3

4) $y = (C_1 + C_2)x^3 + (C_3 + C_4)x^2 + C_5 e^{3x - C_6} + C_5 \sin(C_7 x + C_8)$

order = 6

5) $y = a\sin x + be^{x}$

order = 2

$y' = a\cos x + be^{x}$

$y'' = -a\sin x + be^{x}$

$y + y'' = 2be^{x}$

$y' - y'' = a(\cos x + \sin x)$

$$y = \sin x \frac{(y' - y'')}{(\cos x + \sin x)} + \frac{y + y''}{2}$$

degree = 1

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* Solution of Diff. Eq^n (For 1st order 1st degree)

① Variable separable form

$$\frac{dy}{dx} = \frac{2x}{y}$$

$$\int y\,dy = \int 2x\,dx$$

$$\boxed{y^2 = x^2 + c}$$

Q. Solve foll. diff. eq^n

1. $$\frac{dy}{dx} = x^2 e^y$$

$$\int e^{-y} dy = \int x^2 dx$$

$$-e^{-y} = \frac{x^3}{3} + c$$

$$-\frac{1}{e^y} = \frac{x^3}{3} + c$$

$$-y = -\log\left|-\frac{x^3}{3}\right| + c$$

2. $$\frac{dy}{dx} = e^{y+x} + e^y x^3$$

$$\frac{dy}{dx} = e^y (e^x + x^3)$$

$$\int e^{-y} dy = \int (e^x + x^3)\,dx$$

$$-e^{-y} = e^x + \frac{x^4}{4} + c$$

y = log

3. $\frac{dy}{dx} = x \ln x \operatorname{cosec} y$

$$\int dy \sin y = \int x \ln x \, dx$$

$$-\cos y = \frac{x^2}{2} \ln x - \frac{x^2}{4}$$

4. $\frac{dy}{dx} = \frac{x^2+1}{x(y^2+1)}$

$$\int (y^2+1) \, dy = \int x + \frac{1}{x} \, dx$$

$$\frac{y^3}{3} + y = \frac{x^2}{2} + \ln x + C$$

5. $x\sqrt{1-y^2} \, dx + y\sqrt{1-x^2} \, dy = 0$

$$\frac{dy}{dx} = \frac{x\sqrt{1-y^2}}{-y\sqrt{1-x^2}}$$

$$-\frac{1}{2}\int \frac{2y \, dy}{\sqrt{1-y^2}} = \frac{1}{2}\int \frac{2x \, dx}{\sqrt{1-x^2}}$$

$$+\sqrt{1-y^2} = -\sqrt{1-x^2} + C$$

6. $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

$$\int \sec y \tan y \, dy = \int 2 \sin x \, dx$$

$$\sec y = -2\cos x + C$$

7. $\frac{dy}{dx} = 4\sin x + e^x + y^2 e^x + \frac{y^2}{4} e^x$

$$\frac{dy}{dx} = 4\sin x + e^x \left(1 + y^2 + \frac{y^2}{4}\right)$$

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$\sin x(4+y^2) + e^x\left(1+\frac{y^2}{4}\right)$

$$\left(\sin x + \frac{e^x}{4}\right)(4+y^2) = \frac{dy}{dx}$$

$$\int \sin x + \frac{e^x}{4}\,dx = \int \frac{dy}{4+y^2}$$

$$-\cos x + \frac{e^x}{4} + C = \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right).$$

1. $\sec^2 x \tan y + \sec^2 y \tan x \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{\sec^2 x \tan y}{\sec^2 y \tan x}$$

$$\frac{dy}{dx} = -\frac{1}{\cos x \sin x} \sin y \cos y$$

$$\frac{dy}{dx} = -\frac{\sin 2y}{\sin 2x}$$

$$\int \operatorname{cosec} 2y\,dy = -\int \operatorname{cosec} 2x\,dx$$ ×

$$\ln(\tan x) = -\ln(\tan y) + \log C$$

$$\tan x \tan y = C$$

* Equations reducible to variable separable form:

Ques $\frac{dy}{dx} = (4x+y+1)^2$

$$4x+y+1 = t$$

$$4 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$t^2 = \frac{dt}{dx} - 4 \qquad t^2+4 = \frac{dt}{dx}$$

$$x = \frac{1}{2}\tan^{-1}\left(2x + \frac{y}{2} + \frac{1}{2}\right) + C \qquad x = \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right) + C$$

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$$\frac{dy}{dx} = f(ax+by+c) \quad \underbrace{}_{\to t}$$

Ques $(x+y+1)^2 \, dy = dx$ $\quad y(-1) = 0$

$x+y+1 = t$

$$\frac{dt}{dx} = 1 + \frac{dy}{dx}$$

$$t^2 \left(\frac{dt}{dx} - 1\right) = 1$$

$$t^2 \frac{dt}{dx} = 1 + t^2$$

$$\int \frac{t^2 \, dt}{1+t^2} = \int dx$$

$$x = t - \tan^{-1}(t) + C$$

$$x = (x+y+1) - \tan^{-1}(x+y+1) + C$$

$$y = \tan^{-1}(x+y+1) - 1 + C$$

~~Ques~~

~~$(x+y+1) = t$~~

$$0 = \tan^{-1}(0) - 1 + C$$

$$C = 1$$

$$y = \tan^{-1}(x+y+1)$$

Ques $\frac{dy}{dx} = \sin^2(x+y-8)$

$x+y-8 = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = \sin^2 t$$

$$\frac{dt}{dx} = 1 + \sin^2 t$$

$$\int \frac{dt}{1+2\tan^2 t} = \int dx$$

$$\int \frac{\sec^2 t\, dt}{\tan^2 t + \sec^2 t} = \int dx$$

$$\int \frac{dz}{2z^2+1} = \int dx$$

$$\frac{1}{2}\tan^{-1}(\sqrt{2}z) = x + C$$

$$\frac{1}{2}\tan^{-1}\left(\sqrt{2}(x+y-1)\right) = x + C$$

Ans: $\frac{dy}{dx}\left(\frac{x+y-1}{x+y-3}\right) = \frac{x+y+1}{x+y+3}$

$$x+y-1 = t$$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = \frac{(t+1)(t-3)}{(t+3)(t-1)}$$

$$\frac{dt}{dx} = \frac{t^2-2t-3}{t^2+2t-3} + 1$$

$$\frac{dt}{dx} = \frac{2t^2-6}{t^2+2t-3}$$

$$\frac{dt}{dx} = 2\frac{(t^2-3)}{t^2+2t-3}$$

$$\int \frac{t^2+2t-3}{t^2-3}\, dt = \int 2\, dx$$

$$t + \ln(t^2-3) = 2x + C$$

$$y + \ln\left((x+y)^2-3\right) = x + C$$

06/08/19

II Homogeneous Equation

A function $f(x,y)$ is called a homogeneous function of degree n if $f(\lambda x, \lambda y) = \lambda^n f(x,y)$

$$f(x,y) = 5x^2 + 6xy + y^2$$

$$f(\lambda x, \lambda y) = \lambda^2 5x^2 + \lambda^2 6xy + \lambda^2 y^2$$
$$= \lambda^2 (5x^2 + 6xy + y^2)$$
$$f(\lambda x, \lambda y) = \lambda^2 f(x,y)$$
degree = 2

$$y \frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

$$\frac{dy}{dx} 2xy = x^2+y^2$$

$$dy\, 2xy = x^2+y^2\, dx$$

$$\frac{dx}{dy} = \frac{x^2+y^2}{2xy}$$

$\times 1$

$y = vx$

$$dy = \frac{dv}{dx}$$

$$\frac{dy}{dx} = (v+x)\frac{dv}{dx}$$

$$(v+x)\frac{dv}{dx} = \frac{v^2x^2+x^2}{2x^2v}$$

$$(v+x)\frac{dv}{dx} = \frac{v^2+1}{2v}$$

$$\frac{dv}{dx} = \frac{(v^2+1)}{2v(v+x)}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$\frac{x\,dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$\int \frac{2v}{1-v^2}\,dv = \int \frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$1+v^2 = xc$$

$$1+\frac{y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c$$

Solve diff. eqⁿ:

$$\frac{dy}{dx} - y = x\tan\left(\frac{y}{x}\right) \qquad y(1) = \frac{\pi}{2}$$

$$\frac{dy}{dx} = x\tan\left(\frac{y}{x}\right) + y$$

$$y = vx$$

~~dy =~~

$$\frac{1}{x}\frac{dy}{dx} - \frac{y}{x} = \tan\left(\frac{y}{x}\right)$$

$$\frac{1\,dy}{dx} = v + \frac{dv}{dx}v$$

$$\frac{dy}{dx} = \left(v + x\frac{dv}{dx}\right)$$

$$\left(x\frac{dv}{dx} + v\right) = \tan v$$

$$\sin v = cx$$

$$\sin\frac{y}{x} = cx$$

$$\sin\left(\frac{\pi}{2}\right) = c$$

$$\sin\left(\frac{y}{x}\right) = x \qquad c=1$$

$$(xy+y^2)\,dx + (xy-x^2)\,dy = 0$$

$$xy+y^2\,dx = (x^2-xy)\,dy$$

$$\frac{dy}{dx} = \frac{xy+y^2}{x^2-xy}$$

$$\frac{dy}{dx} = \frac{\frac{y}{x}+\frac{y^2}{x^2}}{1-\frac{y}{x}}$$

$$\frac{y}{x} = v$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{v+v^2}{1-v} - v$$

$$x\frac{dv}{dx} = \frac{v^2+v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{2v^2}{1-v}$$

$$\int \frac{1-v}{2v^2}\,dv = \int \frac{dx}{x}$$

$$\int \frac{1}{2v^2}\,dv - \frac{1}{4}\int \frac{4v}{2v^2} = \int \frac{dx}{x}$$

$$-\frac{1}{2v} - \frac{1}{4}\ln(2v^2) = \ln x + c$$

$$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$$

$$\frac{dy}{dx} - \frac{y}{x} = \sqrt{1+\frac{y^2}{x^2}}$$

$$y = xv$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} - v = \sqrt{1+v^2}$$

$$x\frac{dv}{dx} = \sqrt{1+v^2}$$

$$\int\frac{dv}{\sqrt{1+v^2}} = \int\frac{dx}{x}$$

$$\ln\left(v+\sqrt{1+v^2}\right) = \ln x + \ln C$$

$$v + \sqrt{1+v^2} = Cx$$

$$\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = Cx$$

Ques $x\frac{dy}{dx} = y\left(\log y - \log x + 1\right)$

$$\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$$

$$\frac{y}{x} = v$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v\left(\log v + 1\right)$$

$$x\frac{dv}{dx} = v\log v + v - v$$

$$x\frac{dv}{dx} = v\log v$$

$$x\frac{dv}{dx} = v\log v$$

$$\int \frac{dv}{v\log v} = \int \frac{dx}{x}$$

$$\log(\log v) = \log x + \log c$$

$$\log v = cx$$

$$\log \frac{y}{x} = cx$$

$$x\,dy = \left[y + x\frac{\phi(y/x)}{\phi'\left(\frac{y}{x}\right)} \right] dx$$

$$\frac{dy}{dx} = \left[\frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)} \right]$$

$$v + x\frac{dv}{dx} = \left[v + \frac{\phi(y/x)}{\phi'(y/x)} \right]$$

$$x\frac{dv}{dx} = \frac{\phi(y/x)}{\phi'(y/x)}$$

$$\phi(y/x) = cx$$

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Equations reducible to homogenous equations:

To solve this form

$$\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$$

Case I

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$ then replace $x = X + \alpha$, $y = Y + \beta$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

Choose α and β such that the constant term in Num. & Den. is zero.

eg $$\frac{dy}{dx} = \frac{x - y + 1}{x + 2y + 3}$$

$$\frac{dY}{dX} = \frac{X + \alpha - Y - \beta + 1}{X + \alpha + 2Y + 2\beta + 3}$$

$$\alpha - \beta + 1 = 0$$
$$\alpha + 2\beta = -3$$
$$\alpha = -2/3$$
$$\beta = -5/3$$

$$\frac{dY}{dX} = \frac{X - Y}{X + 2Y}$$

$$\frac{dY}{dX} = \frac{1 - Y/X}{1 + 2Y/X}$$

$$V + X\frac{dV}{dX} = \frac{1 - V}{1 + 2V}$$

$$X\frac{dV}{dX} = \frac{1 - 2V - V - 2V^2}{1 + 2V}$$

$$X\frac{dV}{dX} = \frac{-2V^2 - 2V + 1}{1 + 2V}$$

$$-\int \frac{1 + 2V}{2V^2 + 2V - 1}\, dV = \int \frac{dX}{X}$$

$$-\int \frac{(1 + 2V)\, dV}{(2V + 1)(V - 1)} = \int \frac{dX}{X}$$

$$-\int \frac{1+2V}{2V^2+2V-1} = \int \frac{dX}{X}$$

$$-\frac{1}{2} \ln(2V^2+2V-1) = \log(cX)$$

$$2V^2+2V-1 = (cX)^{-2}$$

$$\frac{2Y^2}{X^2} + \frac{2Y}{X} - 1 = \frac{1}{C^2X^2}$$

$$2Y^2 + 2YX - X^2 = \frac{1}{C^2}$$

$$X = x + 5/3$$

$$Y = y + 9/3$$

Ques $\frac{dy}{dx} = \frac{2x-y+3}{x+2y+4}$

$$x = X+\alpha$$
$$y = Y+\beta$$

$$\frac{dY}{dX} = \frac{2X+2\alpha-Y-\beta+3}{X+\alpha+2Y+2\beta+4}$$

$$\frac{dY}{dX} = \frac{2X-Y}{X+2Y}$$

$$\frac{dY}{dX} = \frac{2-Y/X}{1+2Y/X}$$

$$2\alpha-\beta+3=0$$
$$\alpha+2\beta+4=0$$
$$2\alpha+4\beta+8=0$$
$$5\beta+5=0$$
$$\beta=-1$$
$$\alpha=-2$$

$v + x\,dv$

$$v + x\frac{dv}{dx} = \frac{2-v}{1+2v}$$

$$x\frac{dv}{dx} = \frac{2-v-v-2v^2}{1+v}$$

$$x\frac{dv}{dx} = 2\frac{(1-v-v^2)}{1+2v}$$

$$\int \frac{1+2v}{1-v-v^2} = \int 2\frac{dx}{x}$$

$$\int \frac{1+2v}{v^2+v-1} = -2\int \frac{dx}{x}$$

$$\log(v^2+v-1) = -2\log xC$$

$$v^2+v-1 = \left(\frac{1}{xC}\right)^2$$

$$\frac{y^2}{x^2} + \frac{y}{x} - 1 = \frac{C}{x^2}$$

$$y^2 + yx - x^2 = C$$

$$(y+1)^2 + (y+1)(x+2) - (x+2)^2 = C$$

Case 2

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$

if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

put $a_1x + b_1y = t$
or $a_2x + b_2y = t$

$$\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$$

$$x + y = t$$

~~$dx + dy =$~~ ~~$y = t - x$~~

$x = t - y$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$$\frac{dt}{dx} - 1 = \frac{t+1}{2t+1}$$

$$\frac{dt}{dx} = \frac{1 + t + 2t + 1}{2t+1}$$

$$\frac{dt}{dx} = \frac{3t+2}{2t+1}$$

$$\int \frac{2t+1}{3t+2}\, dt = \int dx$$

$$\frac{2}{3} \int \frac{3t + 3/2}{3t+2}\, dt = \int dx$$

$$\frac{2}{3}\left[t + \frac{1}{2}\int \frac{1\, dt}{3t+2}\right] = x + c$$

$$\frac{2}{3}\left[t + \frac{1}{6}\ln\left(t + 2/3\right)\right] = x + c$$

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$$\frac{2}{3}\left[t + \frac{1}{6}\ln\left(t + 2/3\right)\right] = x + c$$

$$2t + \frac{1}{3}\ln\left(t + 2/3\right) = 3x + c$$

$$6t + \ln\left(t + 2/3\right) = 9x + c$$

III Exact differential equation

$$x\,dy + y\,dx = x^2 dx$$

$$d(xy) = x^2 dx$$

$$xy = \frac{x^3}{3} + c$$

$$y\,dx + x\,dy = y^2 \sin x\,dx$$

$$xy = -y^2 \cos x + c$$

(#)

$$x\,dy + y\,dx = d(xy)$$

$$\frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$$

$$\frac{x\,dy - y\,dx}{x^2} = d\left(\frac{y}{x}\right)$$

$\div y^2$

$$y\,dx - x\,dy = y^2 \sin x\,dx$$

$$\frac{x}{y} = -\cos x + c$$

$$y(1+xy)\,dx - x\,dy = 0$$

$$y\,dx + xy^2 dx = x\,dy$$

$$\frac{dy}{dx} = \frac{1+xy}{x}$$

1

Ques. $\cos x\,dy - y\sin x\,dx = x^2 dx$

$\cos x\,dy = x^2 dx + y\,s.$

$d(y\cos x) = x^2\,dx$

$y\cos x = \frac{x^3}{3} + C$

08/08/19

$d\left(\tan^{-1}\frac{y}{x}\right) = \frac{(x\,dy + y\,dx)}{x^2 + y^2}$

$d\left(\sin^{-1}\frac{xy}{y}\right) = \frac{y}{\sqrt{y^2 - x^2y^2}}\; x\,dy + y\,dx$

* $m(x,y)\,dx + n(x,y)\,dy = 0$

if $\frac{\partial m}{\partial y} = \frac{\partial n}{\partial x}$

then the given eq^n is exact and its sol^n is

$\int m\,dx + \int (\text{terms of } n \text{ not containing } x)\,dy = c$

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(4) $\int (x^3 + 3xy^2)\,dx + (3x^2y + y^3)\,dy = 0$

$6xy \qquad 6xy$

$$\frac{x^4}{4} + \frac{3y^2x^2}{2} + \frac{y^4}{4} = c$$

(5) $(2x\cos y + y^2\cos x)\,dx + (2y\sin x - x^2\sin y)\,dy = 0$

$-2x\sin y + 2y\cos x \qquad 2y\cos x - 2x\sin y$

$$x^2\cos y + \sin x\, y^2 = c$$

(6) $(3xy^2 + x\sin(xy))\,dy + (y^3 + y\sin(xy))\,dx = 0$

$3y^2 + \sin(xy) + xy\cos(xy) \qquad 3y^2 + yx(\cos xy) + \sin(xy)$

$$y^3x - \cos xy = c$$

V Linear Differential Equation :

$\frac{dy}{dx} + Py = Q$ → Linear diff. eq^n of first order first degree

To solve this eq^n first find

Integrating factor → $e^{\int P dx}$

and then the sol^n is

$$\boxed{y e^{\int P dx} = \int Q e^{\int P dx}\, dx}$$

* P & Q are functions of x only or constants

<u>Proof</u>:-

$$\frac{dy}{dx} + Py = Q$$

$$\frac{dy}{dx} e^{\int P dx} + e^{\int P dx} Py = Q e^{\int P dx}$$

$$d(y e^{\int P dx}) = Q e^{\int P dx}$$

$$y e^{\int P dx} = \int Q e^{\int P dx}$$

Ques $\frac{dy}{dx} - y \cot x = \cos x$

$$P = -\cot x \qquad Q = \cos x$$

$$e^{\int -\cot x dx}$$

$$e^{-\log|\sin x|}$$

$$e^{\log|\operatorname{cosec} x|}$$

$$\operatorname{cosec} x$$

$$y \operatorname{cosec} x = \int \cot x$$

$$y \operatorname{cosec} x = +\log|\sin x| + c$$

Ques $x\frac{dy}{dx} + y = x^3$

$$\frac{dy}{dx} + y/x = x^2$$

$$P = \frac{1}{x} \qquad Q = x^2$$

$$e^{\int \frac{1}{x} dx} \qquad e^{\log x} \qquad x$$

$$xy = \int x^3$$

$$xy = \frac{x^4}{4} + c$$

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Ques $\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$

$P = \tan x$

$Q = 2x + x^2 \tan x$

$e^{\int \tan x\, dx}$

$e^{\log|\sec x|}$

$\sec x$

$y\sec x = \int 2x\sec x + x^2 \sec x \tan x \, dx$

~~$y\sec x = \int 2\tan\sec x +$~~

$y\sec x = x^2 \sec x + c$

Ques $(2x - 10y^3)\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} + \frac{y}{2x - 10y^3} = 0$

$\frac{dx}{dy} = \frac{-2x + 10y^3}{y}$

$\frac{dx}{dy} + \frac{2x}{y} = 10y^2$

$\frac{2}{y} = P \qquad Q = 10y^2$

$e^{\int \frac{2}{y} dy} \quad e^{2\log y}$

$xy^2 = 10\int y^4$

$xy^2 = 2y^5 + c$ ~~$xy^2 = 10y^4$~~

* $\frac{dx}{dy} + Px = Q$

$P, Q \rightarrow$ func^n of y | constants

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Ques $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$$\frac{dy}{dx} = \frac{1+y^2}{\tan^{-1}y - y^2}$$

$$\frac{dy}{dx} \quad \frac{dx}{dy} = \frac{\tan^{-1}y - y^2}{1+y^2}$$

$$\frac{dx}{dy} + \frac{\tan^{-1}y}{1+y^2} = \frac{-y^2}{1+y^2}$$

$$e^{\tan^{-1}y}$$

$$x e^{\tan^{-1}y} = \int e^{\tan^{-1}y} \frac{\tan^{-1}y}{1+y^2}\,dy$$

$$x e^{\tan^{-1}y} = e^{\tan^{-1}y} + C$$

Ques $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

$$\frac{dy}{dx} + \frac{2xy}{1-x^2} = \frac{x}{\sqrt{1-x^2}}$$

$$e^{\int \frac{-2x}{1-x^2}dx}$$

$$e^{-\log(1-x^2)}$$

$$\frac{1}{-x^2+1}$$

$$\frac{y}{-x^2+1} = \int \frac{x}{\sqrt{1-x^2}\,(-x^2+1)}\,dx$$

$$-2\frac{y}{-x^2+1} = \int \frac{-2x}{(1-x^2)^{3/2}}$$

$$\frac{-2y}{-x^2+1} = \frac{-2}{\sqrt{1-x^2}} + C$$

$$\frac{y}{x^2-1} = \frac{1}{\sqrt{1-x^2}} + C$$

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*. Reducible to Linear Differential equation (Bernoulli's Equation)

$$\frac{dy}{dx} + px = Qx^n$$

$$\frac{1}{x^n}\frac{dy}{dx} + px^{1-n} = Q$$

Put $t = x^{1-n}$

and then solve to get required equation

Que $\frac{dy}{dx} + \frac{xy}{1-x^2} = x\sqrt{y}$ $[-1 < x < 1]$

$$\frac{1}{\sqrt{y}}\frac{dy}{dx} + \frac{x\sqrt{y}}{1-x^2} = x$$

$\sqrt{y} = t$

$\frac{1}{2\sqrt{y}} y' = \frac{dt}{dx}$

$$\frac{x}{1-x^2}t + 2\frac{dt}{dx} = xt$$

$$\frac{dt}{dx} + \frac{x}{2(1-x^2)} = xt \quad \frac{x}{2}$$

$$\frac{2dt}{dx} + \frac{xt}{1-x^2} = x$$

$$\frac{dt}{dx} + \frac{xt}{2(1-x^2)} = x^2$$

$$e^{-2\times\frac{1}{2}\int\frac{2x}{1-x^2}dx}$$

$$e^{-\frac{1}{4}\log|1-x^2|}$$

$$(1-x^2)^{-\frac{1}{4}}$$

$$\sqrt{y}(1-x^2)^{-1/4} = \frac{1}{2}\int\frac{2x}{(1-x^2)^{\frac{1}{4}}}dx$$

$$\sqrt{y}(1-x)^{-1/4} = \frac{-2}{3}(1-x^2)^{3/4} + C$$

Ans $\frac{dy}{dx} = e^{x-y}(e^x - e^x)$

$$\frac{dy}{dx} = e^{2x-y} - e^x$$

$$\frac{dy}{dx} + e^x = e^{2x-y}$$

$$\frac{dy}{dx} + e^{2x}e^{-y} = -e^x$$

$$e^y \frac{dy}{dx} + e^{x+y} = e^{2x}$$

$$e^{x+y} \qquad e^y = t$$

$$e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + te^x = e^{2x}$$

$$e^{\int e^x dx}$$

$$e^y e^{e^x} = \int e^{e^x + 2x} dx$$

$$e^x = z$$

$$e^{e^x + y} = \int z e^z dz$$

$$e^{e^x + y} = ze^z - \int e^z dz$$

$$e^{e^x + y} = ze^z - e^z + C$$

$$e^{e^x + y} = e^x e^{e^x} - e^{e^x} + C$$

$$e^{\frac{1}{x^3}}$$

$$(xy^2 - y)\, dx + x^2 y\, dy = 0$$

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* Application of Differential Equation

Que. The slope of a curve at any point is reciprocal of twice the ordinate of that point. Find eqⁿ of curve if it passes through (3,4)

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$\int 2y\,dy = \int dx$$

$$y^2 = x + c$$
$$16 = 3 + c$$
$$c = 13$$
$$y^2 = x + 13$$

Que. Find eqⁿ of curve passing through (3,1) and having constant subtangent

$$ym = c$$
$$y\,dy = c\,dx$$
$$\frac{y^2}{2} = cx + c'$$
$$\frac{1}{2} = 3c + c'$$

(crossed out)

$$y\frac{dx}{dy} = k$$

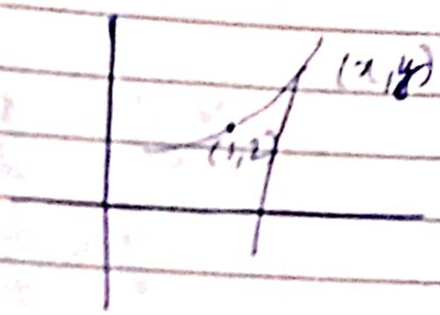
$$\int \frac{dy}{y} = \int \frac{1}{k}dx$$

$$\log y = \frac{x}{k} + c$$

~~c = 3k~~ $c = -3/k$

Find curve passing through (1,2) and x intercept made by any tangent to it is twice the abscissa of point

$(xy^2 - e^{\frac{1}{x^3}})\,dx + x^2y\,dy = 0$



$(Y-y) = \frac{dy}{dx}(X-x)$

$-y = \frac{dy}{dx}(X-x)$

$-y\,dx = x\,dy$

$-\int \frac{1}{2}\frac{dy}{y} = \int \frac{dx}{x}$

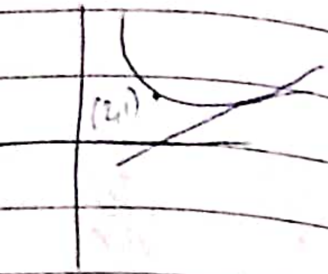
$-\frac{1}{2}\log y = \log x + \log c$

$-\log 2 = \log c$

$c = \frac{1}{2}$

$-\log y = \log x + \log\frac{1}{2}$

Ques Find the curve passing through (2,1) and tangent drawn to it at any point cuts an intercept of 3 units on x-axis.



$$(Y-y) = \frac{dy}{dx}(X-x)$$

$$-y = \frac{dy}{dx}(3-x)$$

$$-y\,dx = 3dy - x\,dy$$

$$-y\,dx = 3dy - x\,dy$$

$$x\,dy - y\,dx = 3dy$$

$$\int \frac{y\,dx - x\,dy}{y^2} = \int \frac{-3dy}{y^2}$$

$$\frac{x}{y} = +\frac{3}{y} + c$$

$$x = 3 + cy$$

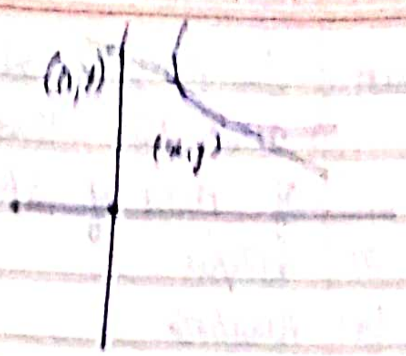
$$2 = 3 + y$$

$$c = -1$$

$$\boxed{x + y = 3}$$

Ques Find eq^n of curve through (1,0) such that any tangent to this curve intersects y-axis at the point which is equidistant from pt of tangency and origin.

$Y - y = \frac{dy}{dx}(X - x)$



$Y = \frac{dy}{dx}(X - x) + y$

$Y^2 = (y - Y)^2 + x^2$

$Y = y/2$

$-y/2 = \frac{dy}{dx}(X - x)$

$Y^2 = y^2 + Y^2 - 2yY + x^2$

$2yY = x^2 + y^2$

$\frac{x^2 + y^2}{2y} = -\frac{dy}{dx}x + y$

$\frac{x^2 + y^2 - 2y^2}{2y} = -x\frac{dy}{dx}$

$\frac{x^2 - y^2}{2y} = -x\frac{dy}{dx}$

$\frac{y^2 - x^2}{2xy} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2}\frac{v^2 - 1}{v}$

$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$

$x\frac{dv}{dx} = -\left(\frac{1 + v^2}{v}\right)$

$\int \frac{2v}{1+v^2}\,dv = \int -\frac{dx}{x}$

$\log(1 + v^2) = -\log x + c$, $c = 0$

$\int v\,dv = -\int \frac{dx}{x}$

$\frac{v^2}{2} = -\log x + c$

$\frac{y^2}{2x^2} = -\log x + c$

$\frac{y^2}{x^2} + \log x = 0$

$1 + \frac{y^2}{x^2} = \frac{c}{x}$

$\boxed{x^2 + y^2 - x = 0}$

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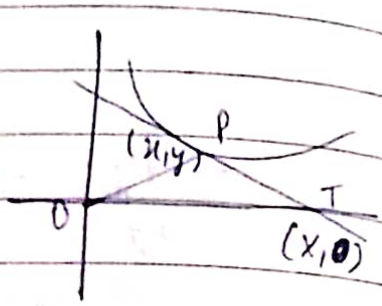
Ques If tangent at any point P on curve meets y-axis at A and line through P ∥ to y-axis meets x-axis at B. If area of ΔAOB is constant then this curve is
(1) Ellipse
(2) Parabola
(3) Circle
(4) Hyperbola
(5) Nothing can be said

Ques If tangent at any point P of a curve meets x-axis at T. Find eqn of curve if radius vector of P = PT



X–

$$(Y-y) = \frac{dy}{dx}(X-x)$$

$$-y = \frac{dy}{dx}(X-x)$$

$$-y\,dx = X\,dy - x\,dy$$

$$x\,dy - y\,dx = X\,dy$$

$$X = x - y\frac{dx}{dy}$$

$$x^2+y^2 = y^2 + \left(y\frac{dx}{dy}\right)^2$$

$$x = y\frac{dx}{dy}$$

$$\int\frac{dy}{y} = \int\frac{dx}{x}$$

$$\log y = \log x + c$$

$$y = cx \quad | \quad yx = c$$

Q. If population grows at the rate of 8% per year how long does it take for that population to double and use diff eq^n.

$$\frac{dP}{dt} = \frac{8P}{100}$$

$$\int \frac{dP}{8P} = \int \frac{dt}{100}$$

$$\frac{1}{8} \log P = \frac{t}{100} + c$$

$$\frac{\log P}{2} = \frac{t}{25} + c$$

$$\frac{\log P_0}{2} = c$$

$$\log P_0 = c$$

$$\log 2P_0 = \frac{t}{25}$$

$$t = \frac{25}{2} \log 2$$

Q. If water at temp 100°C cools down in 10 min to 80°C in a room with temp 25°C. Find
1) temp of water after 20 min.
2) the time after which temp is 40°C.

$$\frac{dT}{dt} = -k(T-S)$$

$$\frac{dT}{dt} = -k(T-25)$$

$$\log(T-25) = -kt + c$$

$$\log 75 = c$$

$$\log 55 = -10k + \log 75$$

$$10k = \log 20$$

$$k = \frac{1}{10} \log 20$$

$$\log(T-25) = \log 75 - 2\log 20$$

$$T - 25 = \frac{75}{400} = \frac{3}{16}$$

$$T = 25 + 0.18$$

$$T = 25.18°C$$

$$\log 15 = -\frac{t}{10} \log 20 + \log 75$$

$$\frac{t}{10} \log 20 = \log 60$$

$$(20)^{t} = (60)^{10}$$

$$t = \frac{10 \log 60}{\log 20}$$

Date

* Orthogonal Trajectory :-

Let a family of curve is represented by an eqⁿ $f(x, y, c) = 0$ then family of curve that intersect members of given family at right angles is called orthogonal trajectory of given family.

Step 1
Form differential eqⁿ representing given family.

Step 2
Replace $\frac{dy}{dx}$ with $-\frac{dx}{dy}$ to obtain diff. eqⁿ representing orthogonal trajectory.

Step 3
Solve this diff. eqⁿ to obtain required family.

Ques Find Orthogonal trajectory for $y^2 = 4ax$

$$\frac{y^2}{x} = 4a$$

$$2yy'x - y^2 = 0$$

$$y^2 = 2yy'x$$

$$\frac{y}{x} = 2\frac{dy}{dx}$$

$$\frac{y}{x} = -2\frac{dx}{dy}$$

$$\int y\,dy = -2\int x\,dx$$

$$\frac{y^2}{2} = -x^2 + c$$

$$\boxed{2x^2 + y^2 = c}$$

Find O.T. for family of curves

$$x^2+y^2=cx$$

$$2x+2yy'=c$$

$$2x+2yy'=\frac{x^2+y^2}{x}$$

$$2yy'=\frac{x^2+y^2-2x^2}{x}$$

$$2y'=\frac{y^2-x^2}{xy}$$

$$y'=\frac{y^2-x^2}{2xy}$$

$$-\frac{dx}{dy}=\frac{1-x^2/y^2}{2x/y}$$

$$\frac{x}{y}=c$$

$$x=cy$$

$$\frac{dx}{dy}=c+y\frac{dc}{dy}$$

$$-c-y\frac{dc}{dy}=\frac{1-c^2}{2c}$$

$$c+y\frac{dc}{dy}=\frac{c^2-1}{2c}$$

$$y\frac{dc}{dy}=\frac{-1-c^2}{2c}$$

$$-\int\frac{2c}{1+c^2}dc=\int\frac{dy}{y}$$

$$-\log|1+c^2|=\log y+\log c$$

$$\frac{1}{1+c^2}=cy$$

$$\frac{y^2}{x^2+y^2}=cy$$

$$x^2+y^2=cy$$

$$\boxed{x^2+y^2+cy=0}$$