

Example 8 Find the range of the following functions given by

(i) $\frac{|x-4|}{x-4}$

(ii) $\sqrt{16-x^2}$

Solution

$$(i) f(x) = \frac{|x-4|}{x-4} = \begin{cases} \frac{x-4}{x-4} = 1, & x > 4 \\ \frac{-(x-4)}{x-4} = -1, & x < 4 \end{cases}$$

Thus the range of $\frac{|x-4|}{x-4} = \{1, -1\}$.

(ii) The domain of f , where $f(x) = \sqrt{16-x^2}$ is given by $[-4, 4]$.

For the range, let $y = \sqrt{16-x^2}$

then $y^2 = 16 - x^2$

or $x^2 = 16 - y^2$

Since $x \in [-4, 4]$

Thus range of $f = [0, 4]$