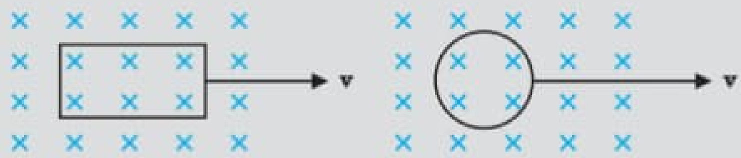


FIGURE 6.8

(d) Predict the polarity of the capacitor in the situation described by Fig. 6.9.

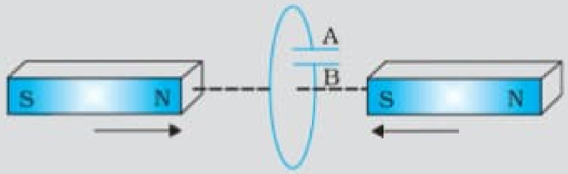


FIGURE 6.9

Solution

(a) No. However strong the magnet may be, current can be induced only by changing the magnetic flux through the loop.

(b) No current is induced in *either* case. Current can not be induced by changing the electric flux.

(c) The induced emf is expected to be constant only in the case of the rectangular loop. In the case of circular loop, the rate of change of area of the loop during its passage out of the field region is not constant, hence induced *emf* will vary accordingly.

(d) The polarity of plate 'A' will be positive with respect to plate 'B' in the capacitor.

EXAMPLE 6.5

6.6 Motional Electromotive Force

Let us consider a straight conductor moving in a uniform and time-independent magnetic field. Figure 6.10 shows a rectangular conductor PQRS in which the conductor PQ is free to move. The rod PQ is moved towards the left with a constant velocity **v** as shown in the figure. Assume that there is no loss of energy due to friction. PQRS forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field **B** which is perpendicular to the plane of this system. If the length RQ = x and RS = l, the magnetic flux Φ_B enclosed by the loop PQRS will be

$$\Phi_B = Blx$$

Since x is changing with time, the rate of change of flux Φ_B will induce an emf given by:

$$\varepsilon = \frac{-\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}(Blx)$$

$$= -Bl\frac{\mathrm{d}x}{\mathrm{d}t} = Blv \qquad (6.5)$$

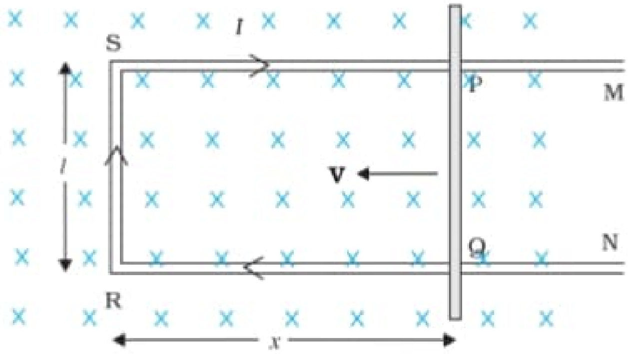


FIGURE 6.10 The arm PQ is moved to the left side, thus decreasing the area of the rectangular loop. This movement induces a current I as shown.

where we have used $dx/dt = -v$ which is the speed of the conductor PQ. The induced emf Blv is called *motional emf*. Thus, we are able to produce induced emf by moving a conductor instead of varying the magnetic field, that is, by changing the magnetic flux enclosed by the circuit.

It is also possible to explain the motional emf expression in Eq. (6.5) by invoking the Lorentz force acting on the free charge carriers of conductor PQ. Consider any arbitrary charge q in the conductor PQ. When the rod moves with speed v, the charge will also be moving with speed v in the magnetic field **B**. The Lorentz force on this charge is qvB in magnitude, and its direction is towards Q. All charges experience the same force, in magnitude and direction, irrespective of their position in the rod PQ. The work done in moving the charge from P to Q is,

$$W = qvBl$$

Since emf is the work done per unit charge,

$$\varepsilon = \frac{W}{q}$$

$$= Blv$$

This equation gives emf induced across the rod PQ and is identical to Eq. (6.5). We stress that our presentation is not wholly rigorous. But it does help us to understand the basis of Faraday's law when the conductor is moving in a uniform and time-independent magnetic field.

On the other hand, it is not obvious how an emf is induced when a conductor is stationary and the magnetic field is changing – a fact which Faraday verified by numerous experiments. In the case of a stationary conductor, the force on its charges is given by

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q\mathbf{E} \qquad (6.6)$$

since $\mathbf{v} = 0$. Thus, any force on the charge must arise from the electric field term **E** alone. Therefore, to explain the existence of induced emf or induced current, we must assume that a time-varying magnetic field generates an electric field. However, we hasten to add that electric fields produced by static electric charges have properties different from those produced by time-varying magnetic fields. In Chapter 4, we learnt that charges in motion (current) can exert force/torque on a stationary magnet. Conversely, a bar magnet in motion (or more generally, a changing magnetic field) can exert a force on the stationary charge. This is the fundamental significance of the Faraday's discovery. Electricity and magnetism are related.

PHYSICS

Interactive animation on motional emf:
http://ngsir.netfirms.com/englishhtm/Induction.htm
http://webphysics.davidson.edu/physlet_resources/bu_semester2/index.html

Example 6.6 A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m, about an axis passing through the centre and perpendicular to the plane of the ring (Fig. 6.11). A constant and uniform magnetic field of 1 T parallel to the axis is present everywhere. What is the emf between the centre and the metallic ring?

EXAMPLE 6.6

Example 6.7
A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field H_E at a place. If H_E = 0.4 G at the place, what is the induced emf between the axle and the rim of the wheel? Note that 1 G = 10^{-4} T.

Solution

Induced emf = $(1/2)\,\omega B R^2$

$$= (1/2) \times 4\pi \times 0.4 \times 10^{-4} \times (0.5)^2$$

$$= 6.28 \times 10^{-5}\ \text{V}$$

The number of spokes is immaterial because the emf's across the spokes are *in parallel*.

EXAMPLE 6.7

6.7 Energy Consideration: A Quantitative Study

In Section 6.5, we discussed qualitatively that Lenz's law is consistent with the law of conservation of energy. Now we shall explore this aspect further with a concrete example.

Let r be the resistance of movable arm PQ of the rectangular conductor shown in Fig. 6.10. We assume that the remaining arms QR, RS and SP have negligible resistances compared to r. Thus, the overall resistance of the rectangular loop is r and this does not change as PQ is moved. The current I in the loop is,

$$I = \frac{\varepsilon}{r}$$

$$= \frac{B l v}{r} \qquad (6.7)$$

On account of the presence of the magnetic field, there will be a force on the arm PQ. This force $I(\mathbf{l} \times \mathbf{B})$, is directed outwards in the direction opposite to the velocity of the rod. The magnitude of this force is,

$$F = I l B = \frac{B^2 l^2 v}{r}$$

where we have used Eq. (6.7). Note that this force arises due to drift velocity of charges (responsible for current) along the rod and the consequent Lorentz force acting on them.

Alternatively, the arm PQ is being pushed with a constant speed v, the power required to do this is,

$$P = F v$$

$$= \frac{B^2 l^2 v^2}{r} \qquad (6.8)$$

The agent that does this work is mechanical. Where does this mechanical energy go? The answer is: it is dissipated as Joule heat, and is given by

$$P_J = I^2 r = \left(\frac{Blv}{r}\right)^2 r = \frac{B^2 l^2 v^2}{r}$$

which is identical to Eq. (6.8).

Thus, mechanical energy which was needed to move the arm PQ is converted into electrical energy (the induced emf) and then to thermal energy.

There is an interesting relationship between the charge flow through the circuit and the change in the magnetic flux. From Faraday's law, we have learnt that the magnitude of the induced emf is,

$$|\varepsilon| = \frac{\Delta \Phi_B}{\Delta t}$$

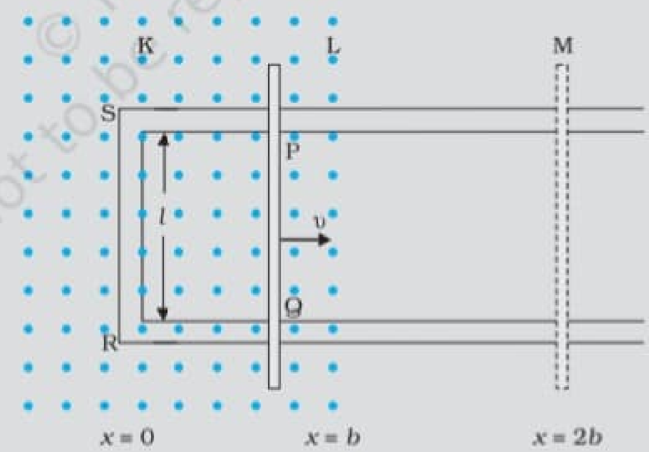
However,

$$|\varepsilon| = Ir = \frac{\Delta Q}{\Delta t} r$$

Thus,

$$\Delta Q = \frac{\Delta \Phi_B}{r}$$

Example 6.8 Refer to Fig. 6.12(a). The arm PQ of the rectangular conductor is moved from $x = 0$, outwards. The uniform magnetic field is perpendicular to the plane and extends from $x = 0$ to $x = b$ and is zero for $x > b$. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from $x = 0$ to $x = 2b$, and is then moved back to $x = 0$ with constant speed v. Obtain expressions for the flux, the induced emf, the force necessary to pull the arm and the power dissipated as Joule heat. Sketch the variation of these quantities with distance.



(a)

FIGURE 6.12

Solution Let us first consider the forward motion from $x = 0$ to $x = 2b$
The flux Φ_B linked with the circuit SPQR is

$$\Phi_B = Blx \qquad 0 \le x < b$$
$$= Blb \qquad b \le x < 2b$$

The induced emf is,

$$\varepsilon = -\frac{d\Phi_B}{dt}$$
$$= -Blv \qquad 0 \le x < b$$
$$= 0 \qquad b \le x < 2b$$

EXAMPLE 6.8