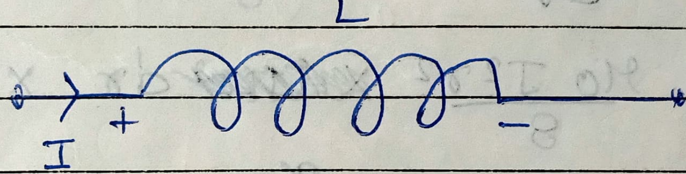
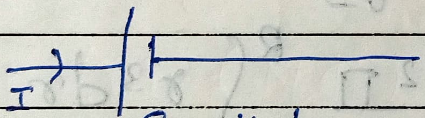
Energy Stored in an Inductor :-



I is increasing from 0 to I



$\mathcal{E} = L\frac{dI}{dt}$

Power consumed by Inductor $= \mathcal{E}I$

Energy consumed (stored) in time dt $= (\mathcal{E}I\,dt)$

$$= \left(L \frac{dI}{dt}\right) I\,dt$$

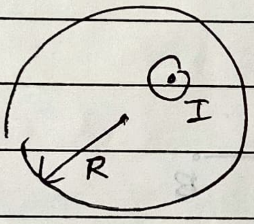
$$= LI\,dt$$

Total Energy ~~stored~~ consumed / stored in inductor $(U) = \int_0^I LI\,dI$

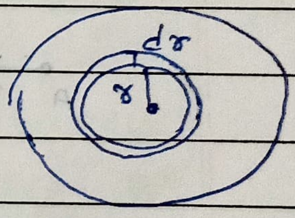
$$\boxed{U = \frac{1}{2} L I^2}$$

Note:- This energy is stored in the magnetic field of inductor. So we can also calculate by $dU/dV = B^2/2\mu_0$

Q:- Find out energy per unit length inside a solid long wire having current density (J) and Radius (R).



$$B = \frac{\mu_0 J r}{2}$$



$$\frac{dU}{dV} = \frac{B^2}{2\mu_0}$$

$$\frac{dU}{dV} = \frac{\mu_0 J^2 r^2}{8}$$

~~dv = ...~~

$dV = 2\pi r L\,dr$

$$dU = \frac{\mu_0 J^2 r^2}{8}\,dr \times 2\pi r L$$

let L be the length of the cylinder

$$\frac{U}{L} = \frac{\mu_0 J^2 \pi}{4} \int_0^R r^3\,dr$$

Energy per unit length $= \frac{\mu_0 J^2 \pi}{4} \times \frac{R^4}{4}$

$$= \frac{\mu_0 J^2 \pi R^4}{16}$$

Q.) The current in a coil of self inductance 2H is increasing according to the law, I = 2 Sin t. Find the amount of energy spent during the period the current changes from 0 A to 2A.

$$\mathcal{E} = L \frac{dI}{dt}$$

$I = 2 \sin t$

$dI = 2 \cos t \, dt$

$$\frac{dU}{dt} = L \frac{dI}{dt} \times I$$

$I = 0 \quad t = 0$

$I = 2 \quad t = \pi/2$

$$U = \int_0^{\pi/2} L \times (2 \sin t)(2 \cos t) \, dt$$

$$U = 4L \int_0^{\pi/2} \sin t \cos t \, dt$$

$$U = 8 \int_0^{\pi/2} \sin t \cos t \, dt$$

$$U = 4 \int_0^{\pi/2} \sin 2t \, dt$$

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$$U = 4 \left[-\frac{\cos 2t}{2} \right]_0^{\pi/2}$$

$$= 4 \left[0 - (-1) \right]$$

$$U = 4$$