Binomial theorem

For $n \in \mathbb{N}$,

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} b + \cdots + {}^nC_{n-1}\, a b^{n-1} + {}^nC_n\, b^n$$

$$= \sum_{k=0}^{n} {}^nC_k\, a^{n-k}\, b^k$$

$$(a-b)^n = \sum_{k=0}^{n} {}^nC_k\, (-1)^k\, a^{n-k}\, b^k$$

1) For $n \in \mathbb{N}$, $0 \leq r < n$, we have

$${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$

2) $$(a+b)^n + (a-b)^n = 2 \sum_{\substack{k=0 \\ k \text{ even}}}^{n} {}^{n}C_{k}\, a^{n-k}\, b^{k}$$

3) $$(a+b)^n - (a-b)^n$$

$$= 2 \sum_{\substack{k=0 \\ k \text{ odd}}}^{n} {}^{n}C_{k}\, a^{n-k}\, b^{k}$$

For $m, n \in \mathbb{N}$, $k \geq 0$

$\sum_{r=0}^{k} {}^nC_r \, {}^mC_{k-r}$ is the co-efficient of x^k in $(1+x)^n(1+x)^m$