Question 24. The first four spectral in the Lyman series of a H-atom are λ= 1218 Å, 1028 Å, 974.3 Å and 951.4 Å. If instead of Hydrogen, we consider deuterium, calculate the shift in the wavelength of these lines.

Solution: Let μ_H and μ_D are the reduced masses of electron for hydrogen and deuterium respectively.

We know that $\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

As n_i and n_f are fixed for by mass series for hydrogen and deuterium.

$$\lambda \propto \frac{1}{R} \text{ or } \frac{\lambda_D}{\lambda_H} = \frac{R_H}{R_D} \qquad \text{...(i)}$$

$$R_R = \frac{m_e e^4}{8\varepsilon_0 ch^3} = \frac{\mu_H e^4}{8\varepsilon_0 ch^3}$$

$$R_D = \frac{m_e e^4}{8\,\varepsilon_0\,ch^3} = \frac{\mu_D e^4}{8\,\varepsilon_0 ch^3}$$

$$\therefore \quad \frac{R_H}{R_D} = \frac{\mu_H}{\mu_D} \qquad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\mu_D} \qquad \text{...(iii)}$$

Reduced mass for hydrogen,

$$\mu_H = \frac{m_e}{1 + m_e/M} \simeq m_e\left(1 - \frac{m_e}{M}\right)$$

Reduced mass for deuterium,

$$\mu_D = \frac{2M \cdot m_e}{2M\left(1 + \frac{m_e}{2M}\right)} \simeq m_e\left(1 - \frac{m_e}{2M}\right)$$

where M is mass of proton

$$\frac{\mu_H}{\mu_D} = \frac{m_e\left(1 - \frac{m_e}{2M}\right)}{m_e\left(1 - \frac{m_e}{2M}\right)} = \left(1 - \frac{m_e}{M}\right)\left(1 - \frac{m_e}{2M}\right)^{-1}$$

$$= \left(1 - \frac{m_e}{M}\right)\left(1 + \frac{m_e}{2M}\right)$$

$$\Rightarrow \quad \frac{\mu_H}{\mu_D} = \left(1 - \frac{m_e}{2M}\right)$$

$$\text{or} \quad \frac{\mu_H}{\mu_D} = \left(1 - \frac{1}{2 \times 1840}\right) = 0.99973 \qquad \text{...(iv)}$$

$(\because \quad M = 1840\, m_e)$

From (iii) and (iv)

$$\frac{\lambda_D}{\lambda_H} = 0.99973, \quad \lambda_D = 0.99973\,\lambda_H.$$

Using $\lambda_H = 1218\,\text{Å}, 1028\,\text{Å}, 974.3\,\text{Å}$ and $951.4\,\text{Å}$, we get

$$\lambda_D = 1217.7\,\text{Å}, 1027.7\,\text{Å}, 974.04\,\text{Å}, 951.1\,\text{Å}$$

Shift in wavelength $(\lambda_H - \lambda_D) \approx 0.3$ Å.