Question 24. A neutron beam of energy E scatters from atoms on a surface with a spacing $d = 0.1$ nm. The first maximum intensity in the reflected beam occurs at $\theta = 30°$. What is the kinetic energy E of the beam in eV?

By the law of conservation of momentum,

$$|p_C| = |p_A| + |p_B|$$

Let us first take the case I when both p_A and p_B are positive,

then $$\lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

In second case when both p_A and p_B are negative,

then $$\lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

In case III when $p_A > 0$, $p_B < 0$ *i.e.*, p_A is positive and p_B is negative,

$$\frac{h}{\lambda_C} = \frac{h}{\lambda_A} - \frac{h}{\lambda_B} = \frac{(\lambda_B - \lambda_A)h}{\lambda_A \lambda_B}$$

$$\Rightarrow \quad \lambda_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A}$$

And in case IV when $p_A < 0$, $p_B > 0$, i.e., p_A is negative and p_B is positive.

$$\therefore \quad \frac{h}{\lambda_C} = \frac{-h}{\lambda_A} + \frac{h}{\lambda_B}$$

$$\Rightarrow \quad = \frac{(\lambda_A - \lambda_B)h}{\lambda_A \lambda_B} \Rightarrow \lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A - \lambda_B}$$

Now, $$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$$

$$= \frac{1}{2} \times \frac{(6.62 \times 10^{-24})^2}{1.67 \times 10^{-27}} \text{J}$$

$$= 0.21 \text{ eV}$$