

Question 9. Relativistic corrections become necessary when the expression for the kinetic energy $\frac{1}{2} mv^2$, becomes comparable with mc^2 . where m is the mass of the particle. At what de-Broglie wavelength, will relativistic corrections become important for an electron?
(a) $\lambda = 10\text{nm}$ (b) $\lambda = 10^{-1}\text{ nm}$ (c) $\lambda = 10^{-4}\text{ nm}$ (d) $\lambda = 10^{-6}\text{ nm}$

Ans-(c),(d)

Key concept: De-Broglie or matter wave is independent of die charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged). The de-Broglie wavelength at which relativistic corrections become important that the phase velocity of the matter waves can be greater than the speed of the light (3×10^8 m/s).

The wavelength of de-Broglie wave is given by

$$\lambda = h/p = h/mv$$

Here, $h = 6.6 \times 10^{-34}$ Js

and for electron, $m = 9 \times 10^{-31}$ kg

To approach these types of problem we use hit and trial method by picking up each option one by one.

In option (a), $\lambda_1 = 10 \text{ nm} = 10 \times 10^{-9} \text{ m} = 10^{-8} \text{ m}$

$$\begin{aligned} \Rightarrow v_1 &= \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-8}} \\ &= \frac{2.2}{3} \times 10^5 \approx 10^5 \text{ m/s} \end{aligned}$$

In option (b), $\lambda_2 = 10^{-1} \text{ nm} = 10^{-1} \times 10^{-9} \text{ m} = 10^{-10} \text{ m}$

$$\Rightarrow v_2 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-10}} \approx 10^7 \text{ m/s}$$

In option (c), $\lambda_3 = 10^{-4} \text{ nm} = 10^{-4} \times 10^{-9} \text{ m} = 10^{-13} \text{ m}$

$$\Rightarrow v_3 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-13}} \approx 10^{10} \text{ m/s}$$

In option (d), $\lambda_4 = 10^{-6} \text{ nm} = 10^{-6} \times 10^{-9} \text{ m} = 10^{-15} \text{ m}$

$$\Rightarrow v_4 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-15}} \approx 10^{12} \text{ m/s}$$

Thus, options (c) and (d) are correct as v_3 and v_4 is greater than 3×10^8 m/s.