Prove that $\boldsymbol{e^{ix} = cos(x) + i\,\sin(x)}$

know that:

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{x} = \sum_{r=0}^{\infty} \frac{x^r}{r!}$$

And $sin\,x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdot \cdots \infty$

$$= \sum_{r=1}^{\infty} \frac{x^{2r-1}}{(2r-1)!} (-1)^{r+1}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots \infty$$

$$= \sum_{r=0}^{\infty} \frac{x^{2r}}{(2r)!} (-1)^{r}$$

let us expand $e^{ix} = 1 + \frac{ix}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} \ldots \ldots . . \ldots$

$$1 + \frac{ix}{1!} - \frac{(x)^2}{2!} - \frac{i(x)^3}{3!} + \frac{(x)^4}{4!} + \frac{i(x)^5}{5!} - \frac{(x)^6}{6!} \ldots \ldots . . \ldots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots \ldots + \mathrm{i}(x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdot \cdots)$$

=cosx + i sinx

Hence proved.