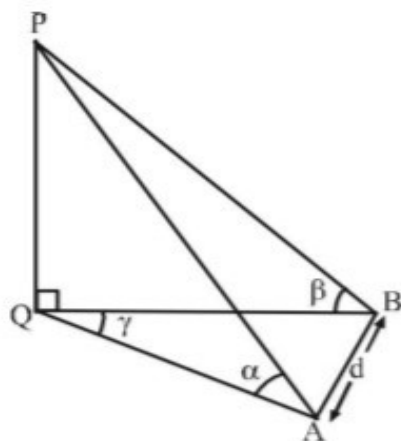8. A vertical pole stands at a point Q on a horizontal ground. A and B are points on the ground, d meters apart. The pole subtends angles α and β at A and B respectively. AB subtends an angle γ at Q. Find the height of the pole.

(1982 - 3 Marks)

Solution: -

8. Let ht. of pole PQ be h.

In $\triangle APQ$, $\tan \alpha = \dfrac{h}{AQ}$

$\Rightarrow \qquad AQ = \dfrac{h}{\tan \alpha}$ \qquad ....(1)

In $\triangle BPQ$, $\tan \beta = \dfrac{h}{BQ} \Rightarrow BQ = \dfrac{h}{\tan \beta}$ \qquad ....(2)

In $\triangle ABQ$, $\cos \gamma = \dfrac{AQ^2 + BQ^2 - AB^2}{2AQ \cdot BQ}$

$\therefore \qquad \cos \gamma = \dfrac{h^2 \cot^2 \alpha + h^2 \cot^2 \beta - d^2}{2h^2 \cot \alpha \cot \beta}$

$\therefore \quad -2h^2 \cot \alpha \cot \beta \cos \gamma + h^2 \cot^2 \alpha + h^2 \cot^2 \beta = d^2$

$\Rightarrow \quad h = \dfrac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta - 2 \cot \alpha \cot \beta \cos \gamma}}$