

6. The sides of a triangle are $\sin \alpha$, $\cos \alpha$ and $\sqrt{1 + \sin \alpha \cos \alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is [2004]
- (a) 150° (b) 90° (c) 120° (d) 60°

Solution: -

6. (c) Let $a = \sin \alpha$, $b = \cos \alpha$ and $c = \sqrt{1 + \sin \alpha \cos \alpha}$
 Clearly a and $b < 1$ but $c > 1$ as $\sin \alpha > 0$ and $\cos \alpha > 0$
 $\therefore c$ is the greatest side and greatest angle is C

$$\begin{aligned} \therefore \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{\sin^2 \alpha + \cos^2 \alpha - 1 - \sin \alpha \cos \alpha}{2 \sin \alpha \cos \alpha} = -\frac{1}{2} \end{aligned}$$

$$\therefore C = 120^\circ$$