7. In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

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(a) 16 (b) 18 (c) 24 (d) 22

Solution: -

7. As the angles A, B, C of ΔABC are in AP

$\therefore$ Let $A = x - d, B = x, C = x + d$

But $A + B + C = 180°$ ($\angle$ Sum prop. of Δ)

$\therefore$ $x - d + x + x + d = 180°$

$\Rightarrow$ $3x = 180° \Rightarrow x = 60° \therefore \angle B = 60°$

Now by sine law in ΔABC, we have

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b}{c} = \frac{\sin B}{\sin C}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin 60°}{\sin C} \qquad \left[\begin{matrix}\text{Using} & b : c = \sqrt{3} : \sqrt{2} \\ \text{and} & \angle B = 60°\end{matrix}\right]$$

$$\Rightarrow \quad \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{2\sin C} \Rightarrow \quad \sin C = \frac{1}{\sqrt{2}} = \sin 45°$$

$\therefore$ $\angle C = 45° \Rightarrow \angle A = 180° - (\angle B + \angle C)$

$= 180° - (60° + 45°) = 75°$