2 . A bird flies in a circle on a horizontal plane. An observer stands at a point on the ground. Suppose 60° and 30° are the maximum and the minimum angles of elevation of the bird and that they occur when the bird is at the points *P* and *Q* respectively on its path. Let θ be the angle of elevation of the bird when it is a point on the arc of the circle exactly midway between *P* and *Q*. Find the numerical value of $\tan^2\theta$.(Assume that the observer is not inside the vertical projection of the path of the bird.) ***(1998 - 8 Marks)***

Solution: -

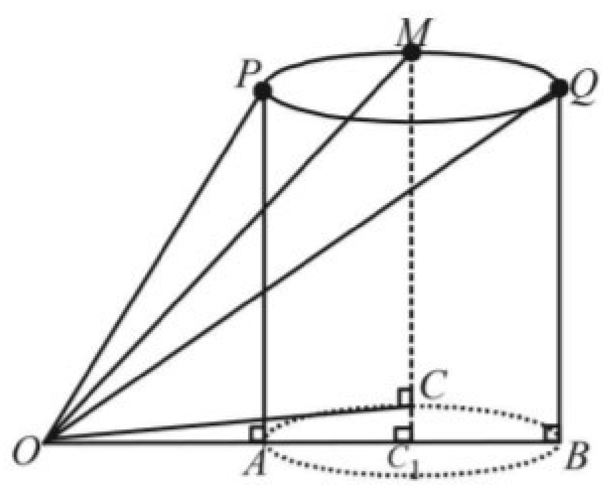
2 . Let *A*, *B* and *C* be the projections of the pts. *P*, *Q* and *M* on the ground.

ATQ, $\angle POA = 60°, \angle QOB = 30°, \angle MOC = \theta$

Let *h* be the ht of circle from ground, then

$AP = CM = BQ = h$

Let $OA = x$ and $AB = d$ (diameter of the projection of the circle on ground with C_1 as centre).



Now in ΔPOA, $\tan 60° = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ....(1)

In ΔQBO, $\tan 30^\circ = \frac{\mathrm{h}}{\mathrm{x+d}} \Rightarrow x + d = h\sqrt{3}$

$$\Rightarrow \qquad d = h\sqrt{3} - \frac{h}{\sqrt{3}} = \frac{2h}{\sqrt{3}} \qquad \text{....(2)}$$

In ΔOMC, $\tan\theta = \frac{h}{OC}$

$$\Rightarrow \quad \tan^2\theta = \frac{h^2}{OC^2} = \frac{h^2}{OC_1^2 + C_1C^2} = \frac{h^2}{\left(x + \frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2}$$

$$= \frac{h^2}{\left(\frac{h}{\sqrt{3}} + \frac{h}{\sqrt{3}}\right)^2 + \left(\frac{h}{\sqrt{3}}\right)^2} \text{ [Using (1) and (2)}$$

$$= \frac{h^2}{\frac{4h^2}{3} + \frac{h^2}{3}} = \frac{3}{5}$$