6. (a) If a circle is inscribed in a right angled triangle ABC with the right angle at B, show that the diameter of the circle is equal to $AB + BC - AC$.

(b) If a triangle is inscribed in a circle, then the product of any two sides of the triangle is equal to the product of the diameter and the perpendicular distance of the third side from the opposite vertex. Prove the above statement. ***(1979)***

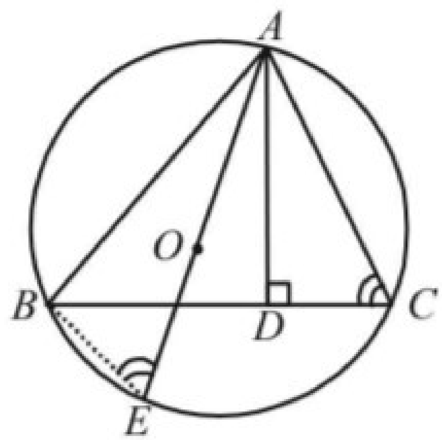
Solution: -

6. **(a)** Inradius of the circle is given by

$$r = (s-b)\tan\frac{B}{2} = \left(\frac{a+b+c}{2} - b\right)\tan\frac{\pi}{4} = \frac{a+c-b}{2}$$

$2r = a + c - b \Rightarrow$ Diameter $= BC + AB - AC$

(b) Given a ΔABC in which $AD \perp BC$, AE is diameter of circumcircle of ΔABC.



To prove :

$AB \times AC = AE \times AD$

Construction : Join BE

Proof : $\angle ABE = 90°$ ($\angle$ in a semi circle)

Now in Δ's ABE and ADC

$\angle ABE = \angle ADC$ (each $90°$)

$\angle AEB = \angle ACD$ ($\angle$'s in the same segment)

$\therefore \quad \Delta ABE \sim \Delta ADC$ (by AA similarity)

$$\Rightarrow \quad \frac{AB}{AD} = \frac{AE}{AC}$$

$\Rightarrow \quad AB \times AC = AD \times AE$ (Proved)