5. With usual notation, if in a triangle ABC;

$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$ then prove that $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$.

(1984 - 4 Marks)

Solution: -

5. Given that, in ΔABC,

$$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$$

where a, b, c are the lengths of sides BC, CA and AB respectively.

Let $$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=k$$

$\Rightarrow$ $b+c=11k$ ....(1)

$c+a=12k$ ....(2)

$a+b=13k$ ....(3)

Adding the above three eqs. we get

$$2(a+b+c)=36k$$

$\Rightarrow$ $a+b+c=18k$ ....(4)

Solving each of (1), (2) and (3) with (4), we get

Now, $\cos A=\frac{b^2+c^2-a^2}{2bc}$