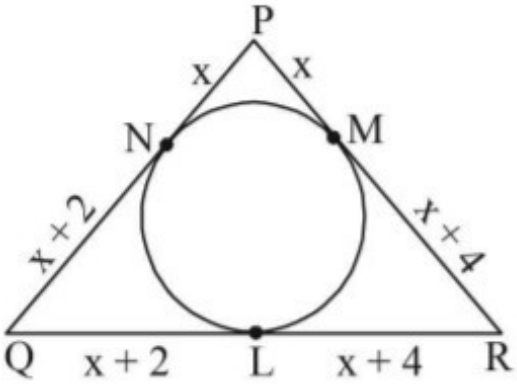
7. In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

(JEE Adv. 2013)

(a) 16 (b) 18 (c) 24 (d) 22

Solution: -

7. ***(b, d)*** Let $PN = x, QL = x+2, RM = x+4$
where x is an even integer.



Then $PM = PN = x, QN = QL = x+2$
and $RL = RM = x+4$
So that $PQ = 2x+2, QR = 2x+6, PR = 2x+4$

$$\text{Now } \cos P = \frac{1}{3} \Rightarrow \frac{PQ^2 + PR^2 - QR^2}{2PQ.PR} = \frac{1}{3}$$

$$\Rightarrow \frac{(2x+2)^2 + (2x+4)^2 - (2x+6)^2}{2.(2x+2).(2x+4)} = \frac{1}{3}$$

$$\Rightarrow 3[(x+1)^2 + (x+2)^2 - (x+3)^2] = 2(x+1)(x+2)$$
$$\Rightarrow 3(x^2-4) = 2(x+1)(x+2) \Rightarrow 3x-6 = 2x+2 \Rightarrow x = 8$$
$$\therefore PQ = 18, QR = 22, PR = 20$$