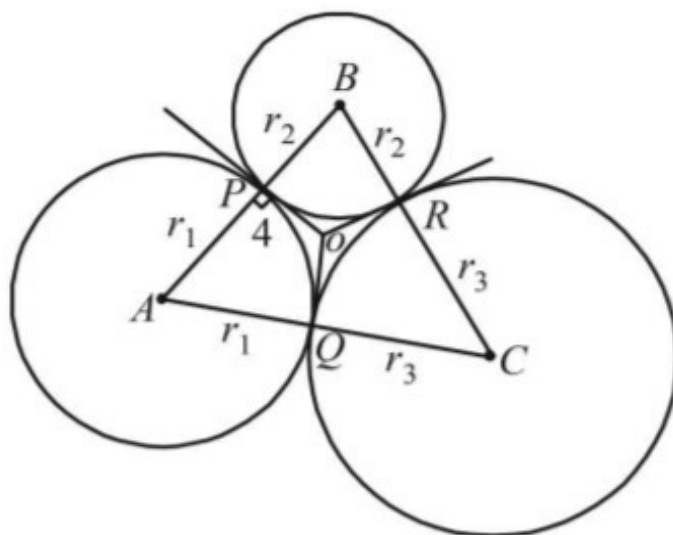


7. Three circles touch the one another externally. The tangent at their point of contact meet at a point whose distance from a point of contact is 4. Find the ratio of the product of the radii to the sum of the radii of the circles. (1992 - 4 Marks)

Solution: -

7. Let us consider three circles with centres at  $A$ ,  $B$  and  $C$  and with radii  $r_1$ ,  $r_2$  and  $r_3$  respectively which touch each other externally at  $P$ ,  $Q$  and  $R$ . Let the common tangents at  $P$ ,  $Q$  and  $R$  meet each other at  $O$ . Then  $OP = OQ = OR = 4$  (given) (lengths of tangents from a pt to a circle are equal).  
Also  $OP \perp AB$ ,  $OQ \perp AC$ ,  $OR \perp BC$ .



$\Rightarrow O$  is the incentre of the  $\Delta ABC$

$$\text{Thus for } \Delta ABC, s = \frac{(r_1 + r_2) + (r_2 + r_3) + (r_3 + r_1)}{2}$$

$$\text{i.e. } s = (r_1 + r_2 + r_3)$$

$$\therefore \Delta = \sqrt{(r_1 + r_2 + r_3) \cdot r_1 \cdot r_2 \cdot r_3} \quad (\text{Heron's formula})$$

$$\text{Now } r = \frac{\Delta}{s}$$

**NOTE THIS STEP:**

$$\Rightarrow 4 = \frac{\sqrt{(r_1 + r_2 + r_3) r_1 r_2 r_3}}{r_1 + r_2 + r_3} \Rightarrow 4 = \frac{\sqrt{r_1 r_2 r_3}}{\sqrt{r_1 + r_2 + r_3}}$$

$$\Rightarrow \frac{r_1 \cdot r_2 \cdot r_3}{r_1 + r_2 + r_3} = \frac{16}{1} \Rightarrow r_1 \cdot r_2 \cdot r_3 : r_1 + r_2 + r_3 = 16 : 1$$