6. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are) *(2010)*

(a) $-(2+\sqrt{3})$ (b) $1+\sqrt{3}$

(c) $2+\sqrt{3}$ (d) $4\sqrt{3}$

Solution: -

6. **(b)** Given that $a = x^2 + x + 1, b = x^2 - 1, c = 2x + 1$

and $\angle C = \pi/6$

Using $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

we get $\cos\dfrac{\pi}{6} = \dfrac{(x^2 + x + 1)^2 + (x^2 - 1)^2 - (2x + 1)^2}{2(x^2 + x + 1)(x^2 - 1)}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{(x^2 + 3x + 2)\,(x^2 - x) + (x^2 - 1)^2}{2(x^2 + x + 1)(x^2 - 1)}$$

$$\Rightarrow \sqrt{3}(x^2 + x + 1)\,(x + 1)(x - 1) = x(x - 1)(x + 1)(x + 2) + (x + 1)^2(x - 1)^2$$

$$\Rightarrow (x + 1)(x - 1)\left[\sqrt{3}(x^2 + x + 1) - x\,(x + 2) - (x + 1)(x - 1)\right] = 0$$

$$\Rightarrow (x + 1)(x - 1)\left[\left(\sqrt{3} - 2\right)x^2 + \left(\sqrt{3} - 2\right)x + \left(\sqrt{3} + 1\right)\right] = 0$$

$$\Rightarrow x = -1, 1, \left(\sqrt{3} + 1\right), -\left(\sqrt{3} + 2\right)$$

Now for $x = -1$ and 1, $b = 0$ which is not possible

Also for $x = -\left(2 + \sqrt{3}\right)$, $c = -4 - 2\sqrt{3} + 1 < 0$ which is not possible

$\therefore\ x = \sqrt{3} + 1$