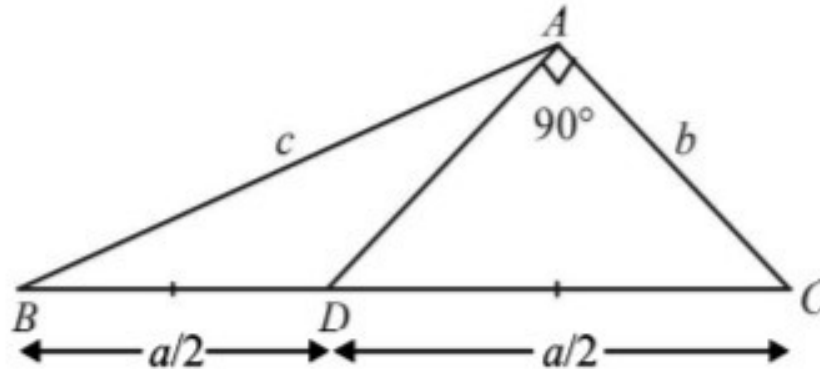**4.** ABC is a triangle. D is the middle point of BC. If AD is perpendicular to AC, then prove that **(1980)**

$$\cos A \cos C = \frac{2(c^2 - a^2)}{3ac}$$

Solution: -

4.

In $\triangle ACD$, $\cos C = \dfrac{b}{a/2} = \dfrac{2b}{a}$ ....(1)

In $\triangle ABC$, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ ....(2)

From (1) and (2),

$$\frac{2b}{a} = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow b^2 = \frac{1}{3}(a^2 - c^2)$$ ....(3)

Also $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\therefore$ $\cos A \cos C = \dfrac{b^2 + c^2 - a^2}{2bc} \times \dfrac{2b}{a} = \dfrac{b^2 + c^2 - a^2}{ac}$

$$= \frac{\frac{1}{3}(a^2 - c^2) + (c^2 - a^2)}{ac} = \frac{2(c^2 - a^2)}{3ac}$$