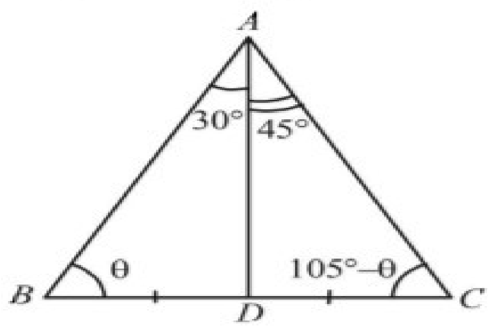
3. In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ *(1985 - 5 Marks)* and it divides the angle A into angles 30° and 45°. Find the length of the side BC.

Solution: -

3. Let AD be the median in ΔABC.
Let $\angle B = \theta$ then $\angle C = 105° - \theta$
In ΔABD, using sine law, we get



$$\frac{BD}{\sin 30°} = \frac{AD}{\sin\theta} \Rightarrow BD = \frac{AD}{2\sin\theta}$$

In ΔACD, using sine law, we get

$$\frac{DC}{\sin 45°} = \frac{AD}{\sin(105° - \theta)} \Rightarrow DC = \frac{AD}{\sqrt{2}\sin(105° - \theta)}$$

As $BD = DC$

$$\Rightarrow \frac{AD}{2\sin 0} = \frac{AD}{\sqrt{2}\sin(105° - 0)}$$

$$\Rightarrow \sin(90° + 15° - \theta) = \sqrt{2}\sin\theta$$

$$\Rightarrow \cos 15° \cos\theta + \sin 15° \sin\theta = \sqrt{2}\sin\theta$$

$$\Rightarrow \cot\theta = \frac{\sqrt{2} - \sin 15°}{\cos 15°} = \frac{5-\sqrt{3}}{\sqrt{3}+1} = 3\sqrt{3} - 4$$

$$\Rightarrow \operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+27+16-24\sqrt{3}}$$

$$\Rightarrow \operatorname{cosec}\theta = 2\sqrt{11-6\sqrt{3}}$$

$$\therefore \quad BD = \frac{AD}{2\sin\theta} = \frac{1}{\sqrt{11-6\sqrt{3}}} \times \frac{2\sqrt{11-6\sqrt{3}}}{2} = 1$$

$$\therefore \quad BC = 2\,BD = 2 \text{ units}$$