

- 1 . Let PQR be a triangle of area Δ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$; where a , b , and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2 \sin P - \sin 2P}{2 \sin P + \sin 2P}$ equals. (2012)

(a) $\frac{3}{4\Delta}$ (b) $\frac{45}{4\Delta}$ (c) $\left(\frac{3}{4\Delta}\right)^2$ (d) $\left(\frac{45}{4\Delta}\right)^2$

Solution: -

- 1 . (c) We have,

$$\frac{2 \sin P - \sin 2P}{2 \sin P + \sin 2P} = \frac{2 \sin P - 2 \sin P \cos P}{2 \sin P + 2 \sin P \cos P} = \frac{1 - \cos P}{1 + \cos P}$$

$$= \frac{2 \sin^2 \frac{P}{2}}{2 \cos^2 \frac{P}{2}} = \tan^2 \frac{P}{2} = \frac{(s-b)(s-c)}{s(s-a)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

$$= \frac{(s-b)^2 (s-c)^2}{s(s-a)(s-b)(s-c)} = \frac{(a+c-b)^2 (a+b-c)^2}{16\Delta^2}$$

$$= \frac{1 \times 9}{16\Delta^2} = \left(\frac{3}{4\Delta}\right)^2$$