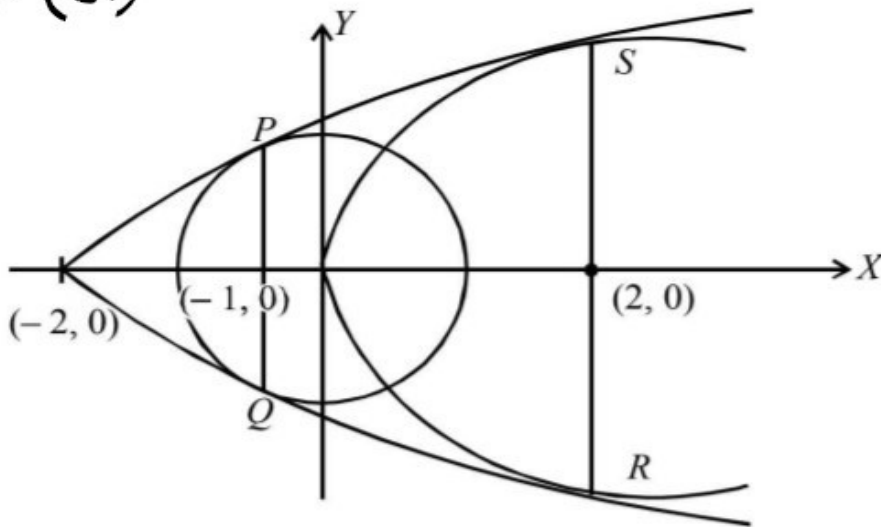8. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral $PQRS$ is

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(a) 3   (b) 6   (c) 9   (d) 15

Solution: -

8. (d)

Let the tangent to $y^2 = 8x$ be $y = mx + \dfrac{2}{m}$

If it is common tangent to parabola and circle, then

$y = mx + \dfrac{2}{m}$ is a tangent to $x^2 + y^2 = 2$

$\therefore \quad \left| \dfrac{\dfrac{2}{m}}{\sqrt{m^2 + 1}} \right| = \sqrt{2} \Rightarrow \dfrac{4}{m^2(1 + m^2)} = 2$

$\Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow (m^2 + 2)(m^2 - 1) = 0$

$\Rightarrow m = 1 \text{ or } -1$

$\therefore$ Required tangents are $y = x + 2$ and $y = -x - 2$

Their common point is $(-2, 0)$

$\therefore$ Tangents are drawn from $(-2, 0)$

$\therefore$ Chord of contact PQ to circle is

$x.(-2) + y.0 = 2$ or $x = -1$

and Chord of contact RS to parabola is

$y.0 = 4(x-2)$ or $x = 2$

Hence coordinates of P and Q are $(-1, 1)$ and $(-1, -1)$

Also coordinates of R and S are $(2, -4)$ and $(2, 4)$

$\therefore$ Area of trapezium $PQRS$ is $\dfrac{1}{2}(2+8) \times 3 = 15$