2. A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B. If AB subtends a right angle at the vertex of the parabola. find the slope of AB.

(1982 - 5 Marks)

Solution: -

2. Parabola $y^2 = 4ax$.

Let at any pt A equation of normal is

$$y = mx - 2am - am^3. \qquad ...(1)$$

Combined equation of OA and OB can be obtained by making equation of parabola homogeneous with the help of normal.

∴ Combined eq. of OA and OB is

$$y^2 = 4ax\left(\frac{mx - y}{2am + am^3}\right)$$

$$\left[\text{ From eqn. (1) using } \frac{mx - y}{2am + am^3} = 1\right]$$

$$y^2 = \frac{4x(mx - y)}{2m + m^3}$$

$$\Rightarrow 4mx^2 - 4xy - (2m + m^3)y^2 = 0$$

But angle between the lines represented by this pair is 90°.

$\Rightarrow$ coeff. of x^2 + coeff of $y^2 = 0 \Rightarrow 4m - 2m - m^3 = 0$

$\Rightarrow m^3 - 2m = 0 \Rightarrow m = 0, \sqrt{2}, -\sqrt{2}$

But for $m = 0$ eq. of normal becomes $y = 0$ which does not intersect the parabola at any other point.

∴ $m = \pm\sqrt{2}$