1. Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point (h, k). Show that $h > 2$. *(1981 - 4 Marks)*

Solution: -

1. The equation of a normal to the parabola $y^2 = 4ax$ in its slope form is given by

$$y = mx - 2am - am^3$$

$\therefore$ Eq. of normal to $y^2 = 4x$, is

$$y = mx - 2m - m^3 \qquad ...(1)$$

Since the normal drawn at three different points on the parabola pass through (h, k), it must satisfy the equation (1)

$\therefore \quad k = mh - 2m - m^3$

$\Rightarrow \quad m^3 - (h-2)\,m + k = 0$

This cubic eq. in m has three different roots say m_1, m_2, m_3

$\therefore \quad m_1 + m_2 + m_3 = 0 \qquad ...(2)$

$m_1 m_2 + m_2 m_3 + m_3 m_1 = -(h-2) \qquad ...(3)$

Now, $(m_1 + m_2 + m_3)^2 = 0 \qquad$ [Squaring (2)]

$\Rightarrow \quad m_1^2 + m_2^2 + m_3^2 = -2\,(m_1 m_2 + m_2 m_3 + m_3 m_1)$

$\Rightarrow \quad m_1^2 + m_2^2 + m_3^2 = 2\,(h-2) \qquad$ [Using (3)]

Since LHS of this equation is he sum of perfect squares, therefore it is + ve

$\therefore \quad h - 2 > 0 \Rightarrow h > 2 \qquad$ Proved