

Notes -

$$\rightarrow \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\rightarrow \tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}, \quad 2x \neq \text{odd multiple of } \frac{\pi}{2}$$

$$\tan(3x) = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

$3x \neq \text{odd multiple of } \frac{\pi}{2}$

$$\rightarrow \cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\cot(x-y) = \frac{1 + \cot x \cot y}{\cot y - \cot x}$$

$$\rightarrow \cot(2x) = \frac{\cot^2 x - 1}{2 \cot x}$$

$$\rightarrow \cot(3x) = \frac{\cot^3 x - 3 \cot x}{3 \cot^2 x - 1}$$

Q) Show that:

$$\frac{\cos 7x + \cos 3x}{\sin 7x - \sin 3x} = \cot 2x$$

Ans) LHS = $\frac{2 \cos \left(\frac{7x+3x}{2} \right) \cos \left(\frac{7x-3x}{2} \right)}{2 \cos \left(\frac{7x+3x}{2} \right) \sin \left(\frac{7x-3x}{2} \right)} = \cot(2x) = \text{RHS}$

Q) Compute $\sin(18^\circ)$.

Ans) $\sin x = \cos \left(\frac{\pi}{2} - x \right)$

$$\sin(36^\circ) = \cos(54^\circ), \quad \theta = 18^\circ$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(3\theta) = 4 \cos^3 \theta - 3 \cos \theta$$

$$2 \sin \theta \cos \theta - (4 \cos^3 \theta - 3 \cos \theta) = 0$$

$$\Rightarrow \cos \theta (2 \sin \theta - 4 \cos^2 \theta + 3) = 0$$

$$\Rightarrow 4 \cos^4 \theta - 2 \sin \theta - 3 = 0$$

$$\Rightarrow 4 - 4 \sin^2 \theta - 2 \sin \theta - 3 = 0$$

$$\Rightarrow 4 \sin^2 \theta + 2 \sin \theta - 1 = 0$$

$$\Rightarrow \sin \theta = \frac{-2 \pm \sqrt{20}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

$$\therefore \sin 18^\circ = \frac{\sqrt{5}-1}{4}$$