6.5 Solution of System of Linear Inequalities in Two Variables

In previous Section, you have learnt how to solve linear inequality in one or two variables graphically. We will now illustrate the method for solving a system of linear inequalities in two variables graphically through some examples.

Example 12 Solve the following system of linear inequalities graphically.

$$x + y \geq 5 \qquad \ldots (1)$$

$$x - y \leq 3 \qquad \ldots (2)$$

Solution The graph of linear equation

$$x + y = 5$$

is drawn in Fig 6.11.

We note that solution of inequality (1) is represented by the shaded region above the line $x + y = 5$, including the points on the line.

On the same set of axes, we draw the graph of the equation $x - y = 3$ as shown in Fig 6.11. Then we note that inequality (2) represents the shaded region above

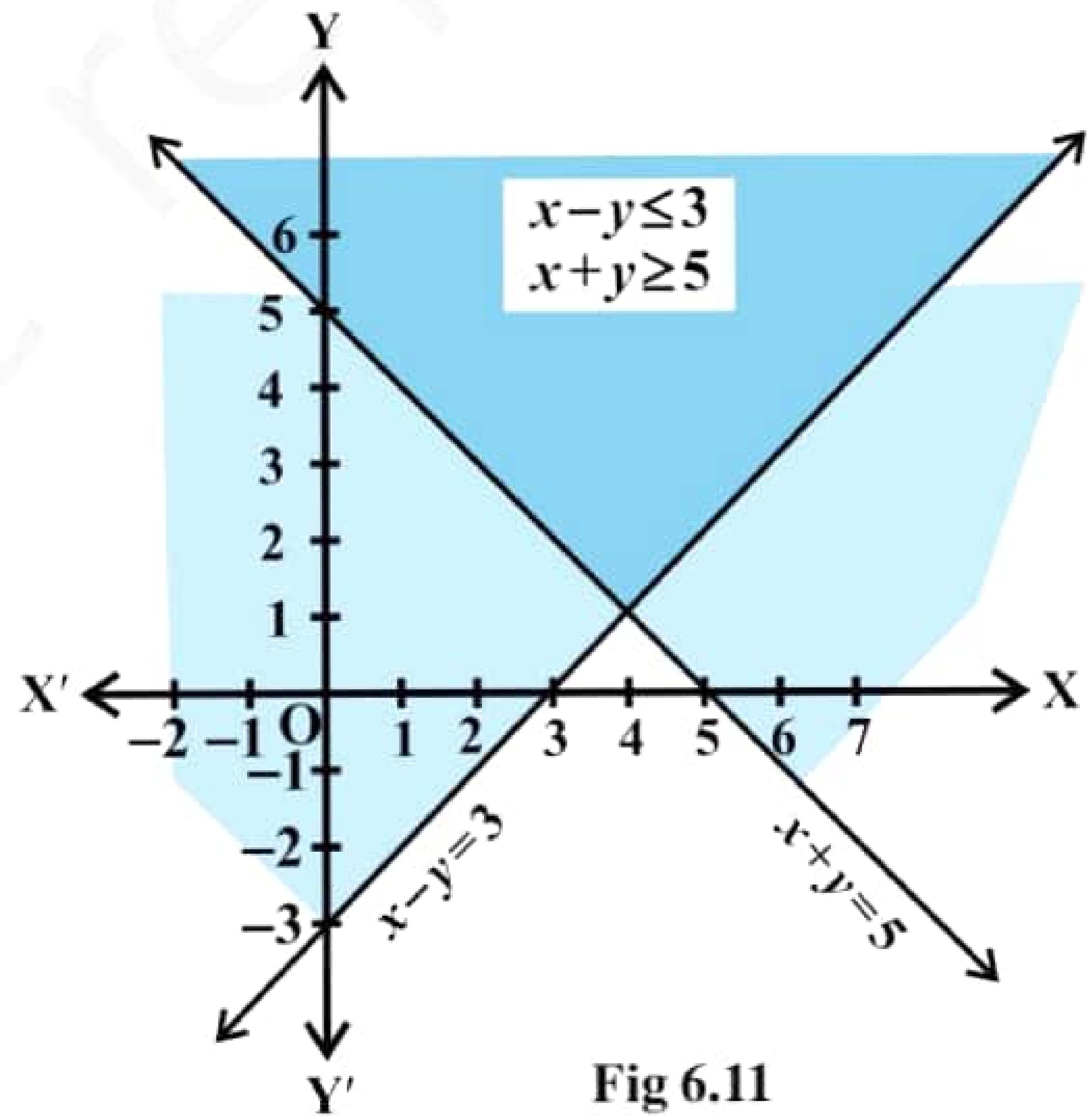

Fig 6.11

the line $x - y = 3$, including the points on the line.

Clearly, the double shaded region, common to the above two shaded regions is the required solution region of the given system of inequalities.

Example 13 Solve the following system of inequalities graphically

$5x + 4y \le 40$ ... (1)

$x \ge 2$ ... (2)

$y \ge 3$ ... (3)

Solution We first draw the graph of the line

$5x + 4y = 40$, $x = 2$ and $y = 3$

Then we note that the inequality (1) represents shaded region below the line $5x + 4y = 40$ and inequality (2) represents the shaded region right of line $x = 2$ but inequality (3) represents the shaded region above the line $y = 3$. Hence, shaded region (Fig 6.12) including all the point on the lines are also the solution of the given system of the linear inequalities.

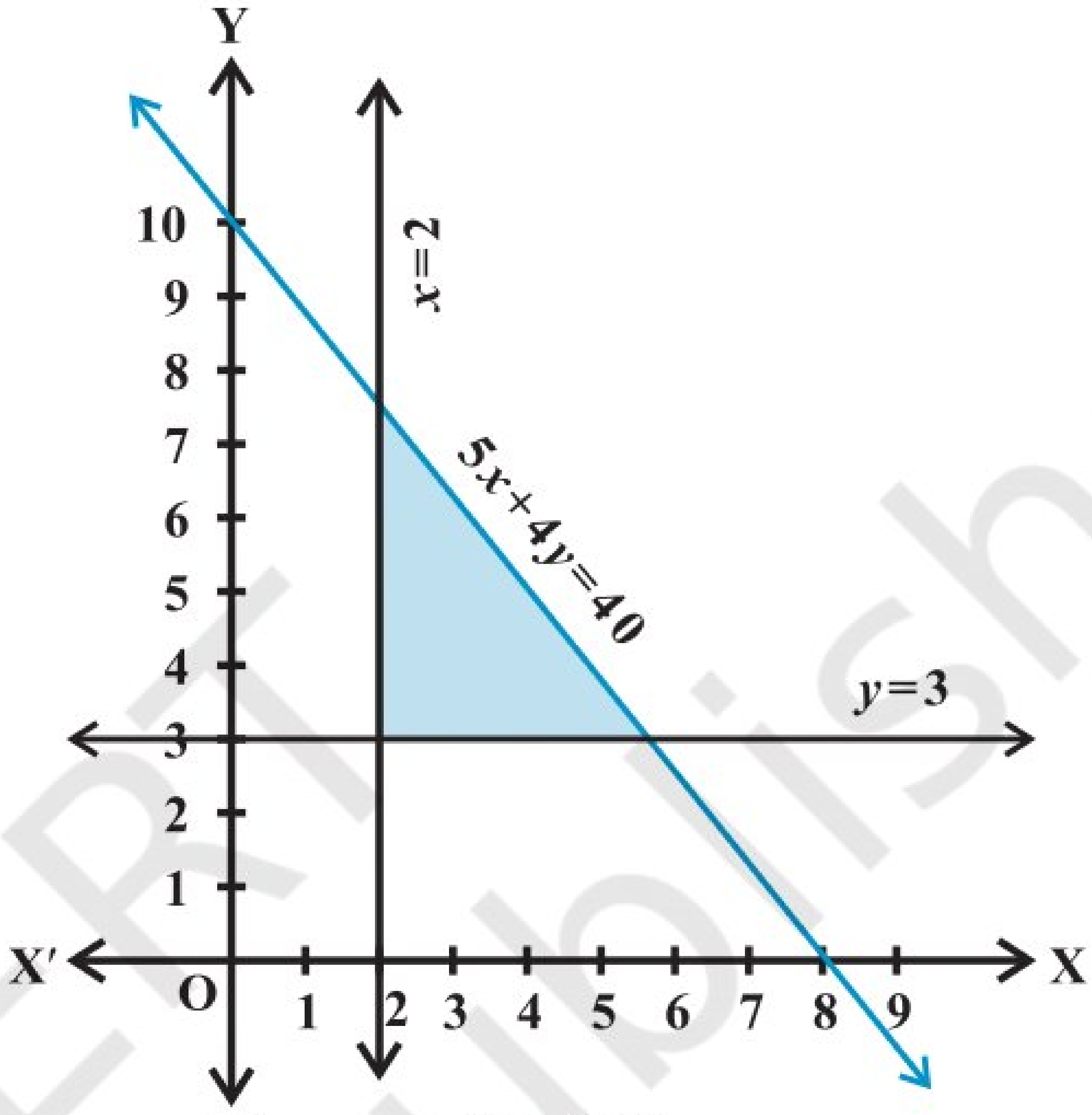

Fig 6.12

In many practical situations involving system of inequalities the variable x and y often represent quantities that cannot have negative values, for example, number of units produced, number of articles purchased, number of hours worked, etc. Clearly, in such cases, $x \ge 0$, $y \ge 0$ and the solution region lies only in the first quadrant.

Example 14 Solve the following system of inequalities

$8x + 3y \le 100$ ... (1)

$x \ge 0$ ... (2)

$y \ge 0$ ... (3)

Solution We draw the graph of the line

$8x + 3y = 100$

The inequality $8x + 3y \le 100$ represents the shaded region below the line, including the points on the line $8x + 3y = 100$ (Fig 6.13).

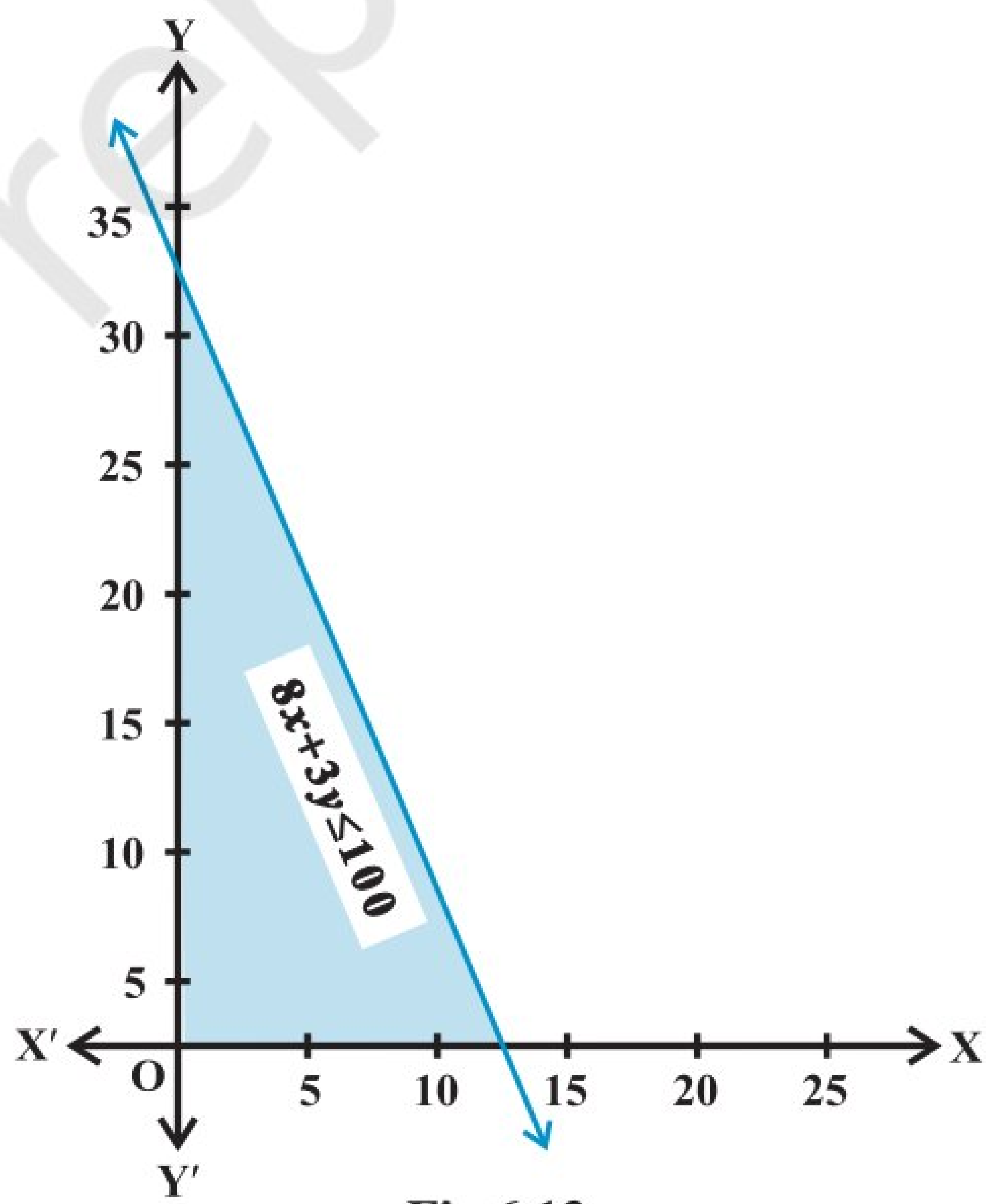

Fig 6.13

Since $x \geq 0$, $y \geq 0$, every point in the shaded region in the first quadrant, including the points on the line and the axes, represents the solution of the given system of inequalities.

Example 15 Solve the following system of inequalities graphically

$x + 2y \leq 8$ ... (1)

$2x + y \leq 8$ ... (2)

$x \geq 0$ ... (3)

$y \geq 0$ ... (4)

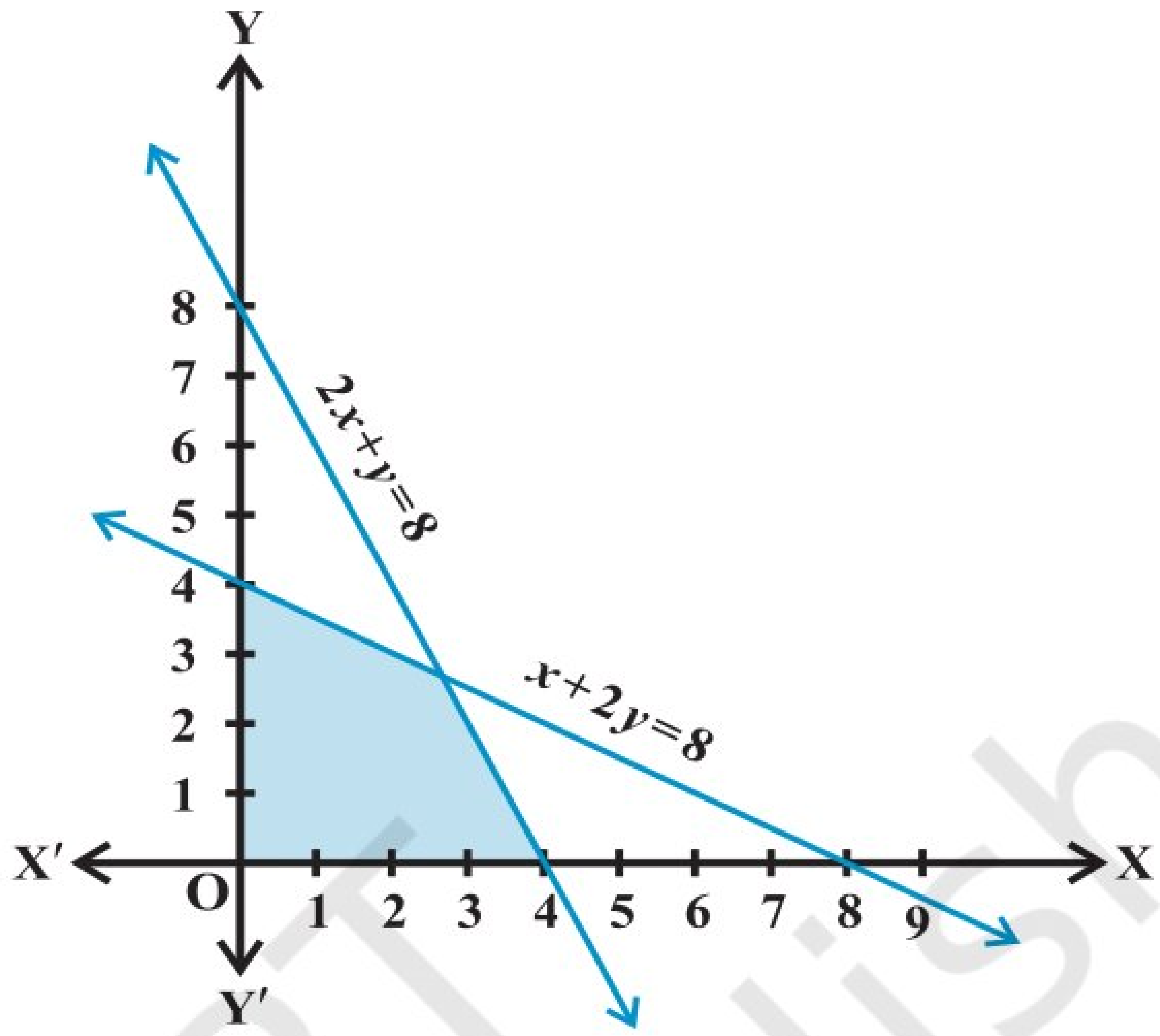

Fig 6.14

Solution We draw the graphs of the lines $x + 2y = 8$ and $2x + y = 8$. The inequality (1) and (2) represent the region below the two lines, including the point on the respective lines.

Since $x \geq 0$, $y \geq 0$, every point in the shaded region in the first quadrant represent a solution of the given system of inequalities (Fig 6.14).

EXERCISE 6.3

Solve the following system of inequalities graphically:

1. $x \geq 3, y \geq 2$
2. $3x + 2y \leq 12,\ x \geq 1, y \geq 2$
3. $2x + y \geq 6, 3x + 4y \leq 12$
4. $x + y \geq 4,\ 2x - y < 0$
5. $2x - y > 1, x - 2y < -1$
6. $x + y \leq 6,\ x + y \geq 4$
7. $2x + y \geq 8,\ x + 2y \geq 10$
8. $x + y \leq 9,\ y > x,\ x \geq 0$
9. $5x + 4y \leq 20,\ \ x \geq 1, y \geq 2$
10. $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0,\ y \geq 0$
11. $2x + y \geq 4,\ x + y \leq 3,\ 2x - 3y \leq 6$
12. $x - 2y \leq 3, 3x + 4y \geq 12,\ x \geq 0, y \geq 1$
13. $4x + 3y \leq 60, y \geq 2x,\ x \geq 3,\ \ x, y \geq 0$
14. $3x + 2y \leq 150, x + 4y \leq 80,\ x \leq 15,\ y \geq 0, x \geq 0$
15. $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$

Miscellaneous Examples

Example 16 Solve $-8 \leq 5x - 3 < 7$.

Solution In this case, we have two inequalities, $-8 \leq 5x - 3$ and $5x - 3 < 7$, which we will solve simultaneously. We have $-8 \leq 5x - 3 < 7$

or $\quad -5 \leq 5x < 10 \quad$ or $\quad -1 \leq x < 2$

Example 17 Solve $-5 \leq \dfrac{5-3x}{2} \leq 8$.

Solution We have $-5 \leq \dfrac{5-3x}{2} \leq 8$

or $\quad -10 \leq 5 - 3x \leq 16 \quad$ or $\quad -15 \leq -3x \leq 11$

or $\quad 5 \geq x \geq -\dfrac{11}{3}$

which can be written as $\dfrac{-11}{3} \leq x \leq 5$

Example 18 Solve the system of inequalities:

$$3x - 7 < 5 + x \qquad \text{... (1)}$$

$$11 - 5x \leq 1 \qquad \text{... (2)}$$

and represent the solutions on the number line.

Solution From inequality (1), we have

$$3x - 7 < 5 + x$$

or $\quad x < 6$

... (3)

Also, from inequality (2), we have

$$11 - 5x \leq 1$$

or $\quad -5x \leq -10 \quad$ i.e., $x \geq 2 \qquad$ ... (4)

If we draw the graph of inequalities (3) and (4) on the number line, we see that the values of x, which are common to both, are shown by bold line in Fig 6.15.

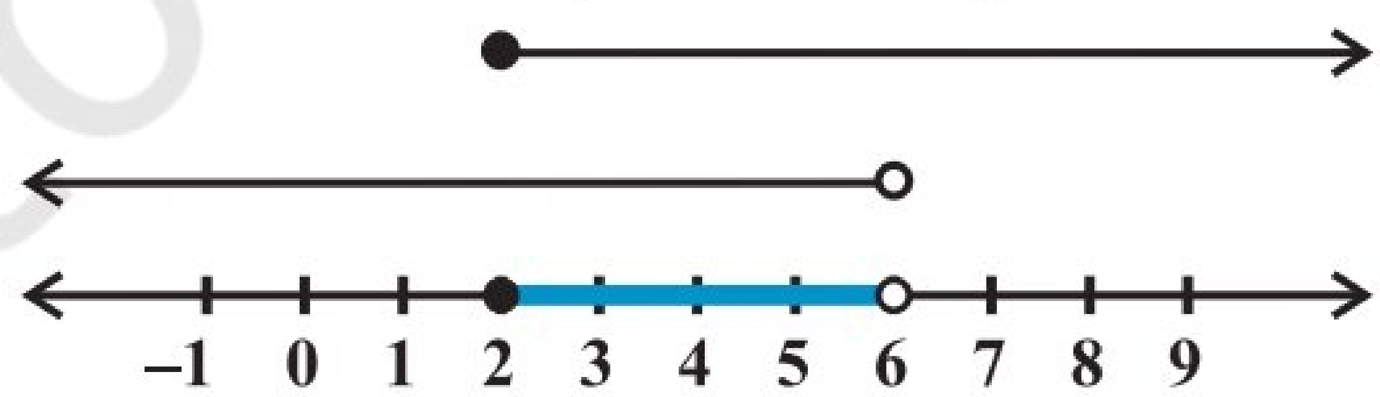

Fig 6.15

Thus, solution of the system are real numbers x lying between 2 and 6 including 2, i.e.,

$$2 \leq x < 6$$

Example 19 In an experiment, a solution of hydrochloric acid is to be kept between 30° and 35° Celsius. What is the range of temperature in degree Fahrenheit if conversion formula is given by $C = \frac{5}{9}(F - 32)$, where C and F represent temperature in degree Celsius and degree Fahrenheit, respectively.

Solution It is given that $30 < C < 35$.

Putting $C = \frac{5}{9}(F - 32)$, we get

$$30 < \frac{5}{9}(F - 32) < 35,$$

or $\frac{9}{5} \times (30) < (F - 32) < \frac{9}{5} \times (35)$

or $54 < (F - 32) < 63$

or $86 < F < 95$.

Thus, the required range of temperature is between 86° F and 95° F.

Example 20 A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Solution Let x litres of 30% acid solution is required to be added. Then

Total mixture = $(x + 600)$ litres

Therefore $30\%\ x + 12\%$ of $600 > 15\%$ of $(x + 600)$

and $30\%\ x + 12\%$ of $600 < 18\%$ of $(x + 600)$

or $\frac{30x}{100} + \frac{12}{100}(600) > \frac{15}{100}(x + 600)$

and $\frac{30x}{100} + \frac{12}{100}(600) < \frac{18}{100}(x + 600)$

or $30x + 7200 > 15x + 9000$

and $30x + 7200 < 18x + 10800$

or $15x > 1800$ and $12x < 3600$

or $x > 120$ and $x < 300$,

i.e. $120 < x < 300$

Thus, the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.

Miscellaneous Exercise on Chapter 6

Solve the inequalities in Exercises 1 to 6.

1. $2 \le 3x - 4 \le 5$

2. $6 \le -3(2x - 4) < 12$

3. $-3 \le 4 - \frac{7x}{2} \le 18$

4. $-15 < \frac{3(x-2)}{5} \le 0$

5. $-12 < 4 - \frac{3x}{-5} \le 2$

6. $7 \le \frac{(3x+11)}{2} \le 11$.

Solve the inequalities in Exercises 7 to 10 and represent the solution graphically on number line.

7. $5x + 1 > -24, \quad 5x - 1 < 24$

8. $2(x - 1) < x + 5, \quad 3(x + 2) > 2 - x$

9. $3x - 7 > 2(x - 6), \quad 6 - x > 11 - 2x$

10. $5(2x - 7) - 3(2x + 3) \le 0, \quad 2x + 19 \le 6x + 47$.

11. A solution is to be kept between 68° F and 77° F. What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by

$$F = \frac{9}{5}C + 32 \,?$$

12. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

13. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

14. IQ of a person is given by the formula

$$IQ = \frac{MA}{CA} \times 100,$$

where MA is mental age and CA is chronological age. If $80 \le IQ \le 140$ for a group of 12 years old children, find the range of their mental age.

Summary

- Two real numbers or two algebraic expressions related by the symbols <, >, ≤ or ≥ form an inequality.
- Equal numbers may be added to (or subtracted from) both sides of an inequality.
- Both sides of an inequality can be multiplied (or divided) by the same positive number. But when both sides are multiplied (or divided) by a negative number, then the inequality is reversed.
- The values of x, which make an inequality a true statement, are called *solutions of the inequality*.
- To represent $x < a$ (or $x > a$) on a number line, put a circle on the number a and dark line to the left (or right) of the number a.
- To represent $x \le a$ (or $x \ge a$) on a number line, put a dark circle on the number a and dark the line to the left (or right) of the number x.
- If an inequality is having ≤ or ≥ symbol, then the points on the line are also included in the solutions of the inequality and the graph of the inequality lies left (below) or right (above) of the graph of the equality represented by dark line that satisfies an arbitrary point in that part.
- If an inequality is having < or > symbol, then the points on the line are not included in the solutions of the inequality and the graph of the inequality lies to the left (below) or right (above) of the graph of the corresponding equality represented by dotted line that satisfies an arbitrary point in that part.
- The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.

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