7. Let P (a sec θ, b tanθ) and Q (a sec ϕ, b tan ϕ), where $\theta + \phi = \pi / 2$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q, then k is equal to *(1999 - 2 Marks)*

(a) $\frac{a^2 + b^2}{a}$ (b) $-\left(\frac{a^2 + b^2}{a}\right)$

(c) $\frac{a^2 + b^2}{b}$ (d) $-\left(\frac{a^2 + b^2}{b}\right)$

Solution: -

7. (d) KEY CONCEPT:

Equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (a sec α, b tan α) is given by $ax \cos \alpha + by \cot \alpha = a^2 + b^2$

Normals at θ, ϕ are $\begin{cases} ax \cos\theta + by \cot\theta = a^2 + b^2 \\ ax \cos\theta + by \cot \phi = a^2 + b^2 \end{cases}$

where $\phi = \frac{\pi}{2} - \theta$ and these pass through (h, k)

$\therefore \quad ah \cos\theta + bk \cot\theta = a^2 + b^2$

$ah \sin\theta + bk \tan\theta = a^2 + b^2$

Eliminating h, bk (cot θ sin θ – tan cos θ)

$= (a^2 + b^2) (\sin\theta - \cos\theta)$ or $k = -(a^2 + b^2)/b$