6. Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle PMN, then the correct expression(s) is(are) *(JEE Adv. 2015)*

(a) $\dfrac{dl}{dx_1} = 1 - \dfrac{1}{3x_1^2}$ for $x_1 > 1$

(b) $\dfrac{dm}{dx_1} = \dfrac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

(c) $\dfrac{dl}{dx_1} = 1 + \dfrac{1}{3x_1^2}$ for $x_1 > 1$

(d) $\dfrac{dm}{dy_1} = \dfrac{1}{3}$ for $y_1 > 0$

Solution: -

6. **(a, b, d)** H : $x^2 - y^2 = 1$ S : Circle with centre $N(x_2, 0)$
Common tangent to H and S at $P(x_1, y_1)$ is

$$xx_1 - yy_1 = 1 \Rightarrow m_1 = \frac{x_1}{y_1}$$

Also radius of circle S with centre N(x_2, 0) through point of contact (x_1, y_1) is perpendicular to tangent

$$\therefore \quad m_1 m_2 = -1 \Rightarrow \frac{x_1}{y_1} \times \frac{0 - y_1}{x_2 - x_1} = -1$$

$$\Rightarrow \quad x_1 = x_2 - x_1 \text{ or } x_2 = 2x_1$$

M is the point of intersection of tangent at P and x-axis

$$\therefore \quad M\left(\frac{1}{x_1}, 0\right)$$

$\because$ Centroid of ΔPMN is (ℓ, m)

$$\therefore \quad x_1 + \frac{1}{x_1} + x_2 = 3\ell \text{ and } y_1 = 3\mathrm{m}$$

Using $x_2 = 2x_1$,

$$\Rightarrow \quad \frac{1}{3}\left(3x_1 + \frac{1}{x_1}\right) = l \text{ and } \frac{y_1}{3} = \mathrm{m}$$

$$\therefore \quad \frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}, \frac{dm}{dy_1} = \frac{1}{3}$$

Also (x_1, y_1) lies on H, $\therefore\ x_1^2 - y_1^2 = 1$ or $y_1 = \sqrt{x_1^2 - 1}$

$$\therefore \quad m = \frac{1}{3}\sqrt{x_1^2 - 1} \quad \therefore \quad \frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$$