2. Let $P(6, 3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at $(9, 0)$, then the eccentricity of the hyperbola is *(2011)*

(a) $\sqrt{\frac{5}{2}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

Solution: -

2. (b) For hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we have

$$\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{b^2x}{a^2y}$$

$\therefore$ Slope of normal at P (6, 3)

$$=-\frac{1}{\left(\frac{dy}{dx}\right)_{(6,3)}}=-\frac{3a^2}{6b^2}$$

$\therefore$ Equation of normal is

$$\frac{y-3}{x-6}=-\frac{3a^2}{6b^2}$$

As it intersects x-axis at (9, 0)

$$\therefore \frac{0-3}{9-6}=\frac{-3a^2}{6b^2} \Rightarrow a^2=2b^2 \qquad ...(1)$$

Also for hyperbola, $b^2 = a^2 (e^2 - 1)$
Using $a^2 = 2b^2$; we get
$b^2 = 2b^2 (e^2 - 1)$

$$\frac{1}{2}=e^2-1 \quad \text{or} \quad e^2=\frac{3}{2} \quad \text{or} \quad e=\sqrt{\frac{3}{2}}$$