

1. Let the eccentricity of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  be

reciprocal to that of the ellipse  $x^2 + 4y^2 = 4$ . If the hyperbola passes through a focus of the ellipse, then (2011)

(a) the equation of the hyperbola is  $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is  $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is  $x^2 - 3y^2 = 3$

**Solution: -**

1. (b, d) For  $x^2 + 4y^2 = 4$  or  $\frac{x^2}{4} + \frac{y^2}{1} = 1$

$$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$$\text{As per question, } \sqrt{1 + \frac{b^2}{a^2}} = \frac{2}{\sqrt{3}} \Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$$

focus of ellipse is  $(\pm\sqrt{3}, 0)$

As hyperbola passes through  $(\pm\sqrt{3}, 0)$

$$\therefore \frac{3}{a^2} = 1 \text{ or } a = \sqrt{3}$$

$\therefore b = 1$  and focus of hyperbola  $(\pm 2, 0)$

$$\therefore \text{Equation of hyperbola is } \frac{x^2}{3} - \frac{y^2}{1} = 1$$

$$\text{or } x^2 - 3y^2 = 3$$