PYQ2

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Question. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of ΔABC is: →

[AIEEE 2003]

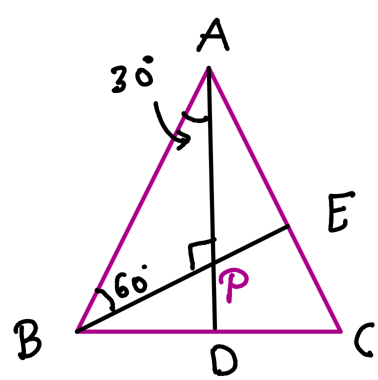
(a) $\frac{8}{3}$

(b) $\frac{16}{3}$

(c) $\frac{32}{3\sqrt{3}}$

(d) $\frac{64}{3}$

Solution.



Given, $\angle DAB = \pi/6$ &

$\angle ABE = \pi/3$

$$\Rightarrow \quad \angle BPA = \pi - \frac{\pi}{6} - \frac{\pi}{3} = \frac{\pi}{2}$$

$\because$ P is centroid of ΔABC

$$\therefore \quad AP = \frac{2}{3} AD = \frac{2}{3} \times 4 = \frac{8}{3} \quad \text{— (1)}$$

In ΔABP,

$$\tan(\angle ABP) = \frac{AP}{BP}$$

$$\Rightarrow \quad BP = \frac{8/3}{\tan 60^\circ} = \frac{8}{3\sqrt{3}} \quad \text{— (2)}$$

$\therefore$ Area of $\Delta ABP = \frac{1}{2} \times AP \times BP$

$$= \frac{1}{2} \times \frac{8}{3} \times \frac{8}{3\sqrt{3}}$$

$$= \frac{32}{9\sqrt{3}} \quad \text{--- } (3)$$

Now, we know that:→

For a ΔABC with centroid G,

$$ar(\Delta GAB) = ar(\Delta GBC) = ar(\Delta GCA) = \frac{1}{3} ar(\Delta ABC)$$

$\therefore$ Area of $\Delta ABC = 3 \times ar(\Delta ABP)$

$$= 3 \times \frac{32}{9\sqrt{3}}$$

$$= \frac{32}{3\sqrt{3}}$$

Ans: option c)