

Q1) $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$ is equal to [2005]

a) $\frac{\log x}{(\log x)^2 + 1} + C$

b) $\frac{x}{x^2 + 1} + C$

c) $\frac{x e^x}{1 + x^2} + C$

d) $\frac{x}{(\log x)^2 + 1} + C$

Solution :-

$$= \int \frac{(\log x - 1)^2}{(1 + (\log x)^2)^2} dx = \int \left[\frac{1}{1 + (\log x)^2} - \frac{2 \log x}{(1 + (\log x)^2)^2} \right] dx$$

Now, put $\log x = t \Rightarrow dx = e^t dt$

$$= \int \left[\frac{e^t}{1 + t^2} - \frac{2t e^t}{(1 + t^2)^2} \right] dt$$

$$= \int e^t \left[\frac{1}{1 + t^2} - \frac{2t}{(1 + t^2)^2} \right] dt$$

$$= \frac{e^t}{1 + t^2} + C = \frac{x}{1 + (\log x)^2} + C$$

Option D