Progression and Series

Finding the n^{th} term of any sequence

i) $1, 3, 5, 7 \ldots \ldots \quad (2n-1)$

ii) $2, 4, 6, 8 \ldots \ldots \quad 2n$

iii) $2, 4, 8, 16 \ldots \ldots \quad 2^n$

iv) $2, 6, 12, 20, \ldots \ldots \quad (n+1)n$

v) $3, 10, 29, 66, \ldots \ldots \quad n^3+2$

vi) $1, -2, 3, -4, 5, -6 \ldots \ldots \quad (-1)^{n-1} \cdot n$

Q. If in any sequence $a_1 = 1$, $a_2 = 1$ such that $a_n = 2a_{n-1} + a_{n-2} \ \forall\ n \geq 3$ find a_6

$a_1 = 1, \quad a_2 = 1$

$a_3 = 2+1 = 3, \quad a_4 = 7, \quad a_5 = 17,$

$\Rightarrow \quad a_6 = 41$

* Arithmatic Progression * →

$a, \quad a+d, \quad a+2d, \ldots \ldots$

$T_1 \quad T_2 \quad T_3 \ldots \ldots$

$\Rightarrow \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = d$

$a =$ first term, $d =$ common difference

* $\boxed{T_n = a + (n-1)d}$

$$T_1' = T_n$$

$$T_2' = T_{n-1}$$

$$T_3' = T_{n-2}$$

$$T_4' = T_{n-3}$$

$$\Rightarrow \ ^{*}\boxed{T_k' = T_{n-k+1}}$$

$a, a+d, a+2d, \ldots \ldots a+(n-1)d$

$T_1+T_n = 2a+(n-1)d,$ $T_2+T_{n-1} = 2a+(n-1)d$

$T_3+T_{n-2} = 2a+(n-1)d, \ldots$

* $T_1+T_2+T_3+\ldots+T_{n-2}+T_{n-1}+T_n$

If $n=$ even

$$\Sigma T_n = \underbrace{\frac{n}{2}}_{\text{no. of pairs}}(2a+(n-1)d$$

If $n=$ odd

$T_1+T_2+\ldots+T_{mid}+\ldots+T_{n-1}+T_n$

$$T_{mid} = \frac{T_1+T_n}{2} = \frac{a+a+(n-1)d}{2} = \frac{2a+(n-1)d}{2}$$

$$S_n = \frac{(n-1)}{2}(2a+(n-1)d) + T_{mid}$$

$$* \boxed{S_n = \frac{n}{2}(2a+(n-1)d)}$$

$$* \boxed{S_n = \frac{(a+T_n)}{2} \cdot n} *$$

Properties of an AP

1) If any constant k is added / − / × / ÷ from each term the resultant series is still in A.P.

2) If 3 terms are in AP

a, b, c

$\Rightarrow b - a = c - b$

$\Rightarrow \boxed{2b = a + c}$

* Geometric Progression *

$a, ar, ar^2, ar^3, \ldots\ldots\ldots ar^{n-1}$

where a = first term of G.P

r = common ratio

$$\boxed{T_n = a r^{n-1}}$$

Q. Consider a G.P. 2, 6, 18, 54, ...

Find i) 20th term.

$$T_{20} = 2 \times (3)^{19}$$

ii) $$\frac{3T_5 + 4T_4 + T_7}{3T_8 + 4T_7 + T_{10}} = \frac{3ar^4 + 4ar^3 + ar^6}{3ar^7 + 4ar^6 + ar^9}$$

$$= \frac{3r + 4 + r^3}{3r^4 + 4r^3 + r^6} = \frac{1}{r^3}$$

(c) The term closest to 500

$ar^{n-1} = 500$

$3^{n-1} = 250$

$n = 6$

Q. If p^{th} term of G.P is q, q^{th} term of that G.P is p. Find $(p+q-1)^{th}$ term.

$a(r)^{p-1} = q$

$a(r)^{q-1} = p$

$\Rightarrow r^{p-q} = \frac{q}{p}$

$a = q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}$

$$\boxed{T_{p+q-1} = q\left(\frac{q}{p}\right)^{\frac{q-1}{p-q}}}$$

* $a, ar, \ldots\ldots, ar^{n-1}$

$S = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$

$rS = ar + ar^2 + \ldots\ldots + ar^{n-1} + ar^n$

$S(1-r) = a - ar^n$

$$\Rightarrow \boxed{S = \frac{a(1-r^n)}{(1-r)}}$$

Q. $1 + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots\ldots$ n terms

Q. $a^n + a^{n-1} \cdot b + a^{n-2} \cdot b^2 + \text{-----} + ab^{n-1} + b^n$

$(n+1)$ term

$$S = \frac{a^n\left(\left(\frac{b}{a}\right)^{n+1} - 1\right)}{\left(\frac{b}{a} - 1\right)} = \frac{b^{n+1} - a^{n+1}}{b-a}$$

* $$\boxed{b^{n+1} - a^{n+1} = (b-a)(a^n + a^{n-1}b + a^{n-2}b^2 + \text{-----} + b^n)}$$

Q. $(1+a) + (1+a+a^2) + (1+a+a^2+a^3) + \text{----} + T_n$

$$\frac{(1-a)\{(1+a) + (1+a+a^2) + (1+a+a^2+a^3) + \text{----} + T_n)}{(1-a)}$$

$$\Rightarrow \frac{(1-a^2) + (1-a^3) + (1-a^4) + \text{------} + T_n}{1-a}$$

$$\Rightarrow \frac{n - a^2(1+a+a^2+ \text{------} n \text{ terms})}{1-a}$$

$$\Rightarrow \frac{1}{(1-a)}\left(n - \frac{a^2(a^n-1)}{(a-1)}\right)$$

$$\Rightarrow \frac{n}{1-a} - \frac{a^2(1-a^n)}{(1-a)^2}$$

$a + ar + ar^2 + \cdots \infty$ terms

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\lim_{n\to\infty} r^n = \begin{cases} \pm\infty & |r|>1 \\ 0 & |r|<1 \end{cases}$$

If $|r| \geq 1$

$S_\infty = \pm\infty$

* If $|r| < 1$

* $\boxed{S_\infty = \frac{a}{1-r}}$

Q. Calcula-

Properties of G.P. :

$T_1, T_2, T_3, T_4, \text{------}, T_n$ are in G.P

1) If we multiple any constant in terms then the new series is also in G.P. i.e.,
$KT_1, KT_2, KT_3, KT_4 \text{------}, KT_n$
are in G.P.

2) If $a_1, a_2, a_3 \text{------} a_n$ are in G.P.
$T_1 a_1, T_2 a_2, T_3 a_3, \text{------} T_n a_n$ is also in G.P.

3) If 3 terms a, b, c are in G.P

$$\frac{b}{a} = \frac{c}{b} \Rightarrow * \boxed{b^2 = ac}$$

4) General terms in G.P

i) 3 terms
$\rightarrow a, ar, ar^2$
$\rightarrow \frac{a}{r}, a, ar$

ii) 4 terms
$\rightarrow \frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
$\rightarrow a, ar^2, ar^3, ar^4$

$c^2 = 4 \Rightarrow \boxed{c = \pm 2}$

5) $a, ar, ar^2 \rightarrow \text{---} \quad ar^{n-2}, ar^{n-1}$

$T_1 \quad T_2 \quad T_3 \qquad T_{n-1} \quad T_n$

$T_1 \cdot T_n = a^2 r^{n-1}$

$T_2 T_{n-1} = a^2 r^{n-1}$

$\Rightarrow \boxed{T_1 T_n = T_2 T_{n-1} = T_3 T_{n-2} = \text{---}}$

* $\prod_{i=0}^{n-1} ar^i = a \cdot ar \cdot ar^2 \text{---} ar^{n-1}$

$= a^n r^{\frac{(n-1)n}{2}}$

$= \left[a^2 r^{(n-1)}\right]^{n/2}$

$$\boxed{(\Sigma a)^2 = \Sigma a^2 + \Sigma ab}$$

* Harmonic Progression *

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$ ———— $\frac{1}{a_n}$ AP

$\Rightarrow$ $a_1, a_2, a_3,$ ———— a_n are in H.P.

$$\boxed{H_n = \frac{1}{a_1+(n-1)d}}$$

Q. If T_4 of HP is $\frac{2}{3}$ and T_{10} of HP is $\frac{2}{11}$ find T_{15}

$$T_4' = 3/2, \quad T_{11}' = \frac{11}{2}$$

$$6d = \frac{8}{2} = 4$$

$$d = \frac{2}{3}$$

$$T_{15}' = \frac{11}{2} + \frac{2}{3}\times 5 = \frac{53}{6}$$

$$T_{15} = \frac{6}{53}$$

Q. If $a_1, a_2, a_3, \ldots, a_n$ are in H.P

P.T $a_1 a_2 + a_2 a_3 + a_3 a_4 + \ldots + a_{n-1} a_n$

$= (n-1)(a_1 a_n)$

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots \frac{1}{a_n}$ are in AP

$\frac{1}{a_1} \to a_1', \quad \frac{1}{a_2} \to a_2', \quad \frac{1}{a_3} \to a_3', \quad \ldots \quad \frac{1}{a_n} \to a_n'$

$\Rightarrow \frac{1}{a_1' a_2'} + \frac{1}{a_2' a_3'} + \frac{1}{a_3' a_4'} + \ldots + \frac{1}{a_{n-1}' a_n'} = ?$

$\Rightarrow \frac{1}{d}\left[\frac{1}{a_n'} + \frac{1}{a_1'}\right] \Rightarrow \frac{1}{d}\left[\frac{a_n' - a_1'}{a_n' a_1'}\right]$

$\Rightarrow \frac{\cancel{d}(n-1)}{\cancel{d}(a_n' a_1')} = (n-1) a_n a_1$