14. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[2011]**

(a) 19 months (b) 20 months
(c) 21 months (d) 18 months

15. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference -2, then the time taken by him to count all notes is **[2010]**

(a) 34 minutes (b) 125 minutes
(c) 135 minutes (d) 24 minutes

16. Let a_1, a_2, a_3........... be terms on A.P. If

$\frac{a_1 + a_2 + ...a_p}{a_1 + a_2 + ...+ a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals **[2006]**

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$
(c) $\frac{2}{7}$ (d) $\frac{11}{41}$

17. If the coefficients of rth, $(r+1)$th, and $(r+2)$th terms in the the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation **[2005]**

(a) $m^2 - m(4r-1) + 4r^2 - 2 = 0$
(b) $m^2 - m(4r+1) + 4r^2 + 2 = 0$
(c) $m^2 - m(4r+1) + 4r^2 - 2 = 0$
(d) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

18. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n,\ m \neq n,\ T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals **[2004]**

(a) $\frac{1}{m} + \frac{1}{n}$ (b) 1
(c) $\frac{1}{mn}$ (d) 0

19. If $1, \log_9 (3^{1-x} + 2), \log_3 (4.3^x - 1)$ are in A.P. then x equals **[2002]**

(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

20. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : **(2016)**

(a) 1 (b) $\frac{7}{4}$
(c) $\frac{8}{5}$ (d) $\frac{4}{3}$

21. Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is a real number. Then the sum $1 + z + z^2 + ... + z^{11}$ is equal to : **(Online April 10, 2016)**

(a) $1365\sqrt{3}i$ (b) $-1365\sqrt{3}i$
(c) $-1250\sqrt{3}i$ (d) $1250\sqrt{3}i$

22. If m is the A.M. of two distinct real numbers l and $n (l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. **(2015)**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$
(c) $4\,l^2mn$ (d) $4\,lm^2n$

23. The sum of the 3^{rd} and the 4^{th} terms of a G.P. is 60 and the product of its first three terms is 1000. If the first term of this G.P. is positive, then its 7^{th} term is : **(Online April 11, 2015)**

(a) 7290 (b) 640
(c) 2430 (d) 320

24. Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is: **[2014]**

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $\sqrt{2}+\sqrt{3}$ (d) $3+\sqrt{2}$

25. The least positive integer n such that

$1 - \frac{2}{3} - \frac{2}{3^2} - .... - \frac{2}{3^{n-1}} < \frac{1}{100}$, is: **[Online April 12, 2014]**

(a) 4 (b) 5
(c) 6 (d) 7

26. In a geometric progression, if the ratio of the sum of first 5 terms to the sum of their reciprocals is 49, and the sum of the first and the third term is 35. Then the first term of this geometric progression is: **[Online April 11, 2014]**

(a) 7 (b) 21
(c) 28 (d) 42

14. **(c)** Let required number of months = n

$\therefore 200 \times 3 + (240 + 280 + 320 + ... + (n-3)^{th} \text{ term})$

$= 11040$

$$\Rightarrow \frac{n-3}{2}\left[2 \times 240 + (n-4) \times 40\right] = 11040 - 600$$

$\Rightarrow (n-3)[240 + 20n - 80] = 10440$

$\Rightarrow (n-3)(20n + 160) = 10440$

$\Rightarrow (n-3)(n+8) = 522$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n+26)(n-21) = 0$

$\therefore n = 21$

15. **(a)** Till 10^{th} minute number of counted notes = 1500

$$3000 = \frac{n}{2}\left[2 \times 148 + (n-1)(-2)\right] = n\left[148 - n + 1\right]$$

$n^2 - 149n + 3000 = 0$

$\Rightarrow n = 125, 24$

But $n = 125$ is not possible

$\therefore$ total time = 24 + 10 = 34 minutes.

16. **(d)** $$\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]}=\frac{p^2}{q^2}$$

$$\Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d}=\frac{p}{q}$$

$$\frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d}=\frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, $p=11, q=41 \Rightarrow \frac{a_6}{a_{21}}=\frac{11}{41}$

17. **(c)** Given ${}^mC_{r-1}, {}^mC_r, {}^mC_{r+1}$ are in A.P.

$$2{}^mC_r={}^mC_{r-1}+{}^mC_{r+1}$$

$$\Rightarrow 2=\frac{{}^mC_{r-1}}{{}^mC_r}+\frac{{}^mC_{r+1}}{{}^mC_r}=\frac{r}{m-r+1}+\frac{m-r}{r+1}$$

$$\Rightarrow m^2-m(4r+1)+4r^2-2=0.$$

18. **(d)** $T_m=a+(m-1)d=\frac{1}{n}$ .....(1)

$T_n=a+(n-1)d=\frac{1}{m}$ .....(2)

$(1)-(2)\Rightarrow (m-n)d=\frac{1}{n}-\frac{1}{m}\Rightarrow d=\frac{1}{mn}$

From (1) $a=\frac{1}{mn}\Rightarrow a-d=0$

19. **(b)** 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P.

$\Rightarrow 2\log_9(3^{1-x}+2)=1+\log_3(4.3^x-1)$

$\Rightarrow \log_3(3^{1-x}+2)=\log_3 3+\log_3(4.3^x-1)$

$\Rightarrow \log_3(3^{1-x}+2)=\log_3[3(4.3^x-1)]$

$\Rightarrow 3^{1-x}+2=3(4.3^x-1)$

$\Rightarrow 3.3^{-x}+2=12.3^x-3.$

Put $3^x=t$

$\Rightarrow \frac{3}{t}+2=12t-3$ or $12t^2-5t-3=0;$

Hence $t=-\frac{1}{3},\frac{3}{4}$

$\Rightarrow 3^x=\frac{3}{4}$ (as $3^x\neq -ve$)

$\Rightarrow x=\log_3\left(\frac{3}{4}\right)$ or $x=\log_3 3-\log_3 4$

$\Rightarrow x=1-\log_3 4$

20. **(d)** Let the GP be a, ar and ar^2 then $a=A+d$; $ar=A+4d$; $ar^2=A+8d$

$$\Rightarrow \frac{ar^2-ar}{ar-a}=\frac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}$$

$r=\frac{4}{3}$

21. **(b)** $z=1+ai$

$z^2=1-a^2+2ai$

$z^2.z=\{(1-a^2)+2ai\}\quad\{1+ai\}$

$=(1-a^2)+2ai+(1-a^2)\quad ai-2a^2$

$\because$ z^3 is real $\Rightarrow 2a+(1-a^2)a=0$

$a(3-a^2)=0\Rightarrow a=\sqrt{3}\ (a>0)$

$$1+z+z^2\,.........\,z^{11}=\frac{z^{12}-1}{z-1}=\frac{(1+\sqrt{3}i)^{12}-1}{1+\sqrt{3}i-1}$$

$$=\frac{(1+\sqrt{3}i)^{12}-1}{\sqrt{3}i}$$

$$(1+\sqrt{3}i)^{12}=2^{12}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{12}$$

$$=2^{12}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{12}=2^{12}(\cos 4\pi+i\sin 4\pi)=2^{12}$$

$$\Rightarrow \frac{2^{12}-1}{\sqrt{3}i}=\frac{4095}{\sqrt{3}i}=-\frac{4095}{3}\sqrt{3}i=-1365\sqrt{3}i$$

22. **(d)** $m=\frac{l+n}{2}$ and common ratio of G.P. $=r=\left(\frac{n}{l}\right)^{\frac{1}{4}}$

$\therefore$ $G_1=l^{3/4}n^{1/4}, G_2=l^{1/2}n^{1/2}, G_3=l^{1/4}n^{3/4}$

$G_1^4+2G^4+G_3^4=l^3n+2l^2n^2+ln^3$

$=ln(l+n)^2$

$=ln\times 2m^2$

$=4lm^2n$

23. **(d)** Let a, ar and ar^2 be the first three terms of G.P

According to the question

$a(ar)(ar^2)=1000\Rightarrow (ar)^3=1000\Rightarrow ar=10$

and $ar^2+ar^3=60\Rightarrow ar(r+r^2)=60$

$\Rightarrow r^2+r-6=0$

$\Rightarrow r=2,-3$

$a=5, a=-\frac{10}{3}$ (reject)

Hence, $T_7=ar^6=5(2)^6=5\times 64=320.$

24. **(b)** Let a, ar, ar^2 are in G.P.

According to the question

$a, 2ar, ar^2$ are in A.P.

$\Rightarrow 2\times 2ar=a+ar^2$

$\Rightarrow \quad 4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$

$$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

Since $r > 1$

$\therefore \quad r = 2 - \sqrt{3}$ is rejected

Hence, $r = 2 + \sqrt{3}$

25. (b) $1 - \frac{2}{3} - \frac{2}{3^2} .... \frac{2}{3^{n-1}} < \frac{1}{100}$

$$\Rightarrow 1 - \frac{2}{3}\left[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ... \frac{1}{3^{n-1}}\right] < \frac{1}{100}$$

$$\Rightarrow \frac{1 - 2\left[\frac{1}{3}\left(\frac{1}{3^n} - 1\right)\right]}{\frac{1}{3} - 1} < \frac{1}{100}$$

$$\Rightarrow 1 - 2\left[\frac{3^n - 1}{2.3^n}\right] < \frac{1}{100}$$

$$\Rightarrow 1 - \left[\frac{3^n - 1}{3^n}\right] < \frac{1}{100}$$

$$\Rightarrow 1 - 1 + \frac{1}{3^n} < \frac{1}{100}$$

$\Rightarrow 100 < 3^n$

Thus, least value of n is 5

26. (c) According to Question

$\Rightarrow \frac{S_5}{S_5'} = 49$ {here, S_5 = Sum of first 5 terms

and S_5 = Sum of their reciprocals)

$$\Rightarrow \frac{\frac{a(r^5 - 1)}{(r-1)}}{\frac{a^{-1}(r^{-5} - 1)}{(r^{-1} - 1)}} = 49$$

$$\Rightarrow \frac{a(r^5 - 1) \times (r^{-1} - 1)}{a^{-1}(r^{-5} - 1) \times (r - 1)} = 49$$

or $$\frac{a^2 (1 - r^5) \times (1 - r) \times r^5}{(1 - r^5) \times (1 - r) \times r} = 49$$

$\Rightarrow a^2 r^4 = 49 \Rightarrow a^2 r^4 = 7^2$

$\Rightarrow \boxed{ar^2 = 7}$ ...(1)

Also, given, $S_1 + S_3 = 35$

$a + ar^2 = 35$ ...(2)

Now substituting the value of eq. (1) in eq. (2)

$a + 7 = 35$

$\boxed{a = 28}$