Hence, the solution of equation is given by

$$y \cdot e^{\frac{-x^2}{2}} = \int (x) \left(e^{\frac{-x^2}{2}}\right) dx + C \quad \text{... (2)}$$

Let $$I = \int (x)\, e^{\frac{-x^2}{2}} dx$$

Let $\frac{-x^2}{2} = t$, then $-x\,dx = dt$ or $x\,dx = -dt$.

Therefore, $I = -\int e^t dt = -e^t = -e^{\frac{-x^2}{2}}$

Substituting the value of I in equation (2), we get

$$y\, e^{\frac{-x^2}{2}} = -e^{\frac{-x^2}{2}} + C$$

or $$y = -1 + C\, e^{\frac{x^2}{2}} \quad \text{... (3)}$$

Now (3) represents the equation of family of curves. But we are interested in finding a particular member of the family passing through (0, 1). Substituting $x = 0$ and $y = 1$ in equation (3) we get

$$1 = -1 + C \cdot e^0 \quad \text{or} \quad C = 2$$

Substituting the value of C in equation (3), we get

$$y = -1 + 2\, e^{\frac{x^2}{2}}$$

which is the equation of the required curve.

EXERCISE 9.6

For each of the differential equations given in Exercises 1 to 12, find the general solution:

1. $\frac{dy}{dx} + 2y = \sin x$

2. $\frac{dy}{dx} + 3y = e^{-2x}$

3. $\frac{dy}{dx} + \frac{y}{x} = x^2$

4. $\frac{dy}{dx} + (\sec x)\, y = \tan x \left(0 \le x < \frac{\pi}{2}\right)$

5. $\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \le x < \frac{\pi}{2}\right)$

6. $x\frac{dy}{dx} + 2y = x^2 \log x$

7. $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$

8. $(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx \ (x \ne 0)$

9. $x\frac{dy}{dx} + y - x + xy\cot x = 0 \;\; (x \neq 0)$ **10.** $(x+y)\frac{dy}{dx} = 1$

11. $y\,dx + (x - y^2)\,dy = 0$ **12.** $(x + 3y^2)\frac{dy}{dx} = y \;\; (y > 0)$.

For each of the differential equations given in Exercises 13 to 15, find a particular solution satisfying the given condition:

13. $\frac{dy}{dx} + 2y\tan x = \sin x;\ y = 0$ when $x = \frac{\pi}{3}$

14. $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2};\ y = 0$ when $x = 1$

15. $\frac{dy}{dx} - 3y\cot x = \sin 2x;\ y = 2$ when $x = \frac{\pi}{2}$

16. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

17. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

18. The Integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(A) e^{-x} (B) e^{-y} (C) $\frac{1}{x}$ (D) x

19. The Integrating Factor of the differential equation

$$(1 - y^2)\frac{dx}{dy} + yx = ay \;(-1 < y < 1) \text{ is}$$

(A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$ (C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$

Miscellaneous Examples

Example 24 Verify that the function $y = c_1 e^{ax}\cos bx + c_2 e^{ax}\sin bx$, where c_1, c_2 are arbitrary constants is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)\,y = 0$$