

* TIDAL FORCES, ENERGY CONSERVATION

- Estimation of distances of Astronomical bodies from the Earth.
- The Radius of the Earth.
- Estimating the sizes of the Moon, the Sun and the plane.

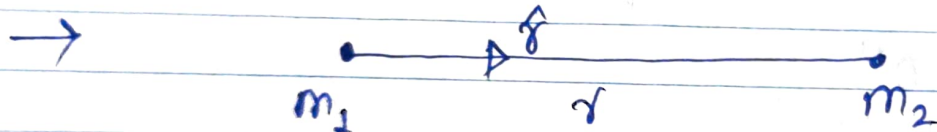
* Galilean law of falling bodies

acceleration is independent of the mass of the falling body.

$$ma = mg \quad [a = g]$$

Inertial mass is indistinguishable from gravitational mass

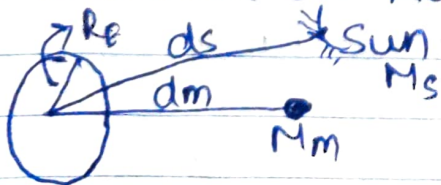
$$m_I = m_G$$



$$\vec{F}_{1 \rightarrow 2} = -\frac{G m_1 m_2}{r^2} \hat{r}$$

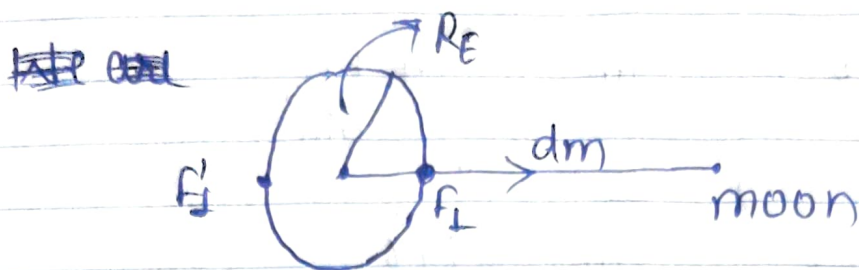
$$\vec{F}_{2 \rightarrow 1} = -\vec{F}_{1 \rightarrow 2}$$

* Phenomena of Tides:-



For any point on the surface of the Earth the true distance varies b/w $(ds \pm R_E)$

* Moon - Earth Forces



$$F_L = \frac{G M_m M_{dm}}{(d - R_E)^2}$$

$$\Delta F_L = F_L - F'_L$$

$$F'_L = \frac{G M_E M_{dm}}{(d + R_E)^2}$$

$$\boxed{d \gg R_E}$$

$$F_L = \frac{K}{(d - R_E)^2} \quad K = G M_m M_{dm}$$

$$= \frac{K}{d^2 \left[1 - \frac{R_E}{d} \right]^2}$$

$$= F_0 \left[\frac{1}{1 - \left(\frac{2R_E}{d} - \frac{R_E^2}{d^2} \right)} \right]$$

$$\boxed{\frac{R_E}{d} \ll 1}$$

$$= F_0 \frac{1}{1 - \alpha}$$

$$\frac{1}{1 - \alpha} = 1 + \alpha + \alpha^2 + \dots (O(\alpha^3))$$

$$\alpha = \left(\frac{2R_E}{d} - \frac{R_E^2}{d^2} \right); \quad \frac{R_E}{d} = \gamma$$

$$= (2\gamma - \gamma^2) \alpha$$

$$= 1 + 2\gamma + 3\gamma^2$$

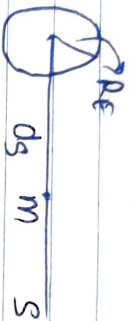
$$F_1 = F_0 [1 + 2x + 3x^2] + \text{higher order terms}$$

$$F_1 = \frac{k}{(dm + R_E)^2} = \frac{k}{dm^2} \left(\frac{1}{1 + \frac{R_E^2}{dm^2}} \right)$$

$$\Rightarrow \frac{k}{dm^2} (1 + x)^{-2}$$

$$\Delta F_1 = 4F_0 \left(\frac{R_E}{dm} \right) \quad \text{gives a dimensionless quantity}$$

$\Delta F_2 \Rightarrow$ Force on the Earth due to the Sun at diametrically opposite position.



$$\Delta F_2 = 4F_0 \left(\frac{R_E}{ds} \right) \quad ; \quad \Delta F_2 = \frac{4GM_E M_S}{dm} \cdot \frac{R_E}{ds}$$

$$\Delta F_2 = \frac{4GM_E}{ds^3} \cdot \frac{R_E}{ds}$$

* We calculate ratio

$$\frac{\Delta F_1}{\Delta F_2} = \frac{M_m}{M_s} \left(\frac{ds}{dm} \right)^3 \approx 3.5$$

$$\left. \begin{aligned} M_{\text{moon}} &= 7.3 \times 10^{22} \text{ kg} \\ M_s &= 2 \times 10^{30} \text{ kg} \\ ds &= 1.50 \times 10^8 \text{ km} \end{aligned} \right\}$$

$$dm = 0.3 \times 10^6 \text{ kg}$$

* Gravitational potential Energy



$$E_T = \frac{1}{2} m v^2 \quad \frac{1}{2} k x^2$$

$$\frac{dE}{dt} = 0 \Rightarrow m v \frac{dv}{dt} + k x \frac{dx}{dt} = 0 \quad \Rightarrow \text{const.}$$