$$g(h) = \frac{F(h)}{m} = \frac{GM_E}{(R_E + h)^2}. \qquad (8.14)$$

This is clearly less than the value of g on the surface of earth : $g = \frac{GM_E}{R_E^2}$. For $h << R_E$, we can expand the RHS of Eq. (8.14) :

$$g(h) = \frac{GM_E}{R_E^2(1 + h/R_E)^2} = g\,(1 + h/R_E)^{-2}$$

For $\frac{h}{R_E} << 1$, using binomial expression,

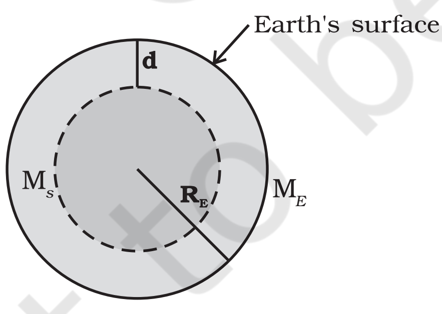
$$g(h) \cong g\left(1 - \frac{2h}{R_E}\right). \qquad (8.15)$$

Equation (8.15) thus tells us that for small heights h above the value of g decreases by a factor $(1 - 2h/R_E)$.

Now, consider a point mass m at a depth d below the surface of the earth (Fig. 8.8(b)), so that its distance from the centre of the earth is $(R_E - d)$ as shown in the figure. The earth can be thought of as being composed of a smaller sphere of radius $(R_E - d)$ and a spherical shell of thickness d. The force on m due to the outer shell of thickness d is zero because the result quoted in the previous section. As far as the smaller sphere of radius $(R_E - d)$ is concerned, the point mass is outside it and hence according to the result quoted earlier, the force due to this smaller sphere is just as if the entire mass of the smaller sphere is concentrated at the centre. If M_s is the mass of the smaller sphere, then,

$$M_s / M_E = (R_E - d)^3 / R_E^3 \qquad (8.16)$$

Since mass of a sphere is proportional to be cube of its radius.



(b)

Fig. 8.8 (b) *g at a depth d. In this case only the smaller sphere of radius (R_E–d) contributes to g.*

Thus the force on the point mass is

$$F(d) = G\, M_s\, m \,/\, (R_E - d)^2 \qquad (8.17)$$

Substituting for M_s from above , we get

$$F(d) = G\, M_E\, m\,(R_E - d) \,/\, R_E^3 \qquad (8.18)$$

and hence the acceleration due to gravity at a depth d,

$g(d) = \frac{F(d)}{m}$ is

$$g(d) = \frac{F(d)}{m} = \frac{GM_E}{R_E^3}(R_E - d)$$

$$= g\frac{R_E - d}{R_E} = g(1 - d/R_E) \qquad (8.19)$$

Thus, as we go down below earth's surface, the acceleration due gravity decreases by a factor $(1 - d/R_E)$. The remarkable thing about acceleration due to earth's gravity is that it is maximum on its surface decreasing whether you go up or down.

8.7 GRAVITATIONAL POTENTIAL ENERGY

We had discussed earlier the notion of potential energy as being the energy stored in the body at its given position. If the position of the particle changes on account of forces acting on it, then the change in its potential energy is just the amount of work done on the body by the force. As we had discussed earlier, forces for which the work done is independent of the path are the conservative forces.

The force of gravity is a conservative force and we can calculate the potential energy of a body arising out of this force, called the gravitational potential energy. Consider points close to the surface of earth, at distances from the surface much smaller than the radius of the earth. In such cases, the force of gravity is practically a constant equal to mg, directed towards the centre of the earth. If we consider a point at a height h_1 from the surface of the earth and another point vertically above it at a height h_2 from the surface, the work done in lifting the particle of mass m from the first to the second position is denoted by W_{12}

$$W_{12} = \text{Force} \times \text{displacement}$$
$$= mg\,(h_2 - h_1) \qquad (8.20)$$

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If we associate a potential energy $W(h)$ at a point at a height h above the surface such that

$$W(h) = mgh + W_o \quad (8.21)$$

(where W_o = constant) ;
then it is clear that

$$W_{12} = W(h_2) - W(h_1) \quad (8.22)$$

The work done in moving the particle is just the difference of potential energy between its final and initial positions.Observe that the constant W_o cancels out in Eq. (8.22). Setting $h = 0$ in the last equation, we get $W(h = 0) = W_o$. $h = 0$ means points on the surface of the earth. Thus, W_o is the potential energy on the surface of the earth.

If we consider points at arbitrary distance from the surface of the earth, the result just derived is not valid since the assumption that the gravitational force mg is a constant is no longer valid. However, from our discussion we know that a point outside the earth, the force of gravitation on a particle directed towards the centre of the earth is

$$F = \frac{G M_E m}{r^2} \quad (8.23)$$

where M_E = mass of earth, m = mass of the particle and r its distance from the centre of the earth. If we now calculate the work done in lifting a particle from $r = r_1$ to $r = r_2$ $(r_2 > r_1)$ along a vertical path, we get instead of Eq. (8.20)

$$W_{12} = \int_{r_1}^{r_2} \frac{G M m}{r^2} \mathrm{d}r$$

$$= -G M_E m \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \quad (8.24)$$

In place of Eq. (8.21), we can thus associate a potential energy $W(r)$ at a distance r, such that

$$W(r) = -\frac{G M_E m}{r} + W_1 , \quad (8.25)$$

valid for $r > R$,

so that once again $W_{12} = W(r_2) - W(r_1)$. Setting r = infinity in the last equation, we get W (r = infinity) = W_1. Thus, W_1 is the potential energy at infinity. One should note that only the difference of potential energy between two points has a definite meaning from Eqs. (8.22) and (8.24). One conventionally sets W_1 equal to zero, so that the potential energy at a point is just the amount of work done in displacing the particle from infinity to that point.

We have calculated the potential energy at a point of a particle due to gravitational forces on it due to the earth and it is proportional to the mass of the particle. The gravitational potential due to the gravitational force of the earth is defined as the potential energy of a particle of unit mass at that point. From the earlier discussion, we learn that the gravitational potential energy associated with two particles of masses m_1 and m_2 separated by distance by a distance r is given by

$$V = -\frac{G m_1 m_2}{r} \quad \text{(if we choose } V = 0 \text{ as } r \to \infty\text{)}$$

It should be noted that an isolated system of particles will have the total potential energy that equals the sum of energies (given by the above equation) for all possible pairs of its constituent particles. This is an example of the application of the superposition principle.

Example 8.3 Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also obtain the potential at the centre of the square.

Answer Consider four masses each of mass m at the corners of a square of side l; See Fig. 8.9. We have four mass pairs at distance l and two diagonal pairs at distance $\sqrt{2}\, l$

Hence,

$$W(r) = -4\frac{G m^2}{l} - 2\frac{G m^2}{\sqrt{2}\, l}$$

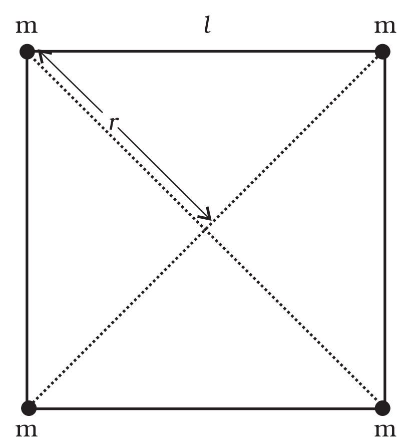

Fig. 8.9

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