If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)\,dx$ is equal to

a) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + c$ b) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

c) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + c$ d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + c$

Solution:-

$\int f(x)dx = \psi(x)$

Let $x^3 = t \Rightarrow 3x^2dx = dt$

$\Rightarrow \int x^5 f(x^3)dx = \frac{1}{3}\int t f(t)\,dt$

$= \frac{1}{3}\left[t\int f(t)dt - \int\left\{1\cdot\int f(t)dt\right\}dt\right]$

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

option (B)