Question1

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Question 1 In ΔABC, if $a=5$, $b=4$, $c=3$, find the circum-radius of ΔGAB, where G is the centroid of ΔABC.

(a) $\frac{6\sqrt{3}}{9}$

(b) $\frac{4\sqrt{3}}{7}$

(c) $\frac{5\sqrt{13}}{12}$

(d) $\frac{8\sqrt{7}}{5}$

Solution:→

Length of median AD

$$= \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$$

$$= \frac{1}{2}\sqrt{2\times 16+2\times 9-25}$$

$$= \frac{5}{2} \quad \text{—— } (1)$$

Now, we know that the "Centroid divides the median in the ratio of 2:1.

$$\therefore \quad AG = \frac{2}{3}AD = \frac{2}{3}\times\frac{5}{2} = \frac{5}{3} \quad \text{—— } (2)$$

Now, for given triangle,

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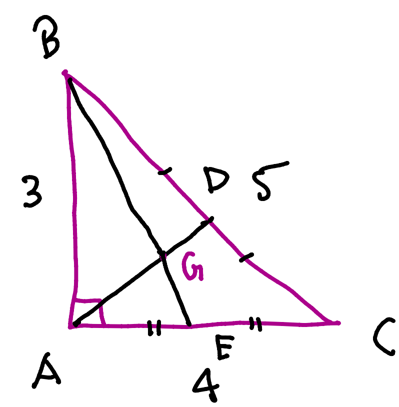
$$a^2 = b^2 + c^2$$

$\therefore$ ΔABC is right angle triangle with $\angle A = 90^\circ$.



Now, $BE = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$

$$= \frac{1}{2}\sqrt{2 \times 5^2 + 2 \times 3^2 - 4^2}$$

$$= \frac{1}{2}\sqrt{52} = \sqrt{13} \quad \text{—— } (3)$$

$\therefore$ $BG = \frac{2}{3} BE = \frac{2}{3}\sqrt{13}$

Now,

For a ΔABC with centroid G,

area of ΔGAB = area of ΔGBC = area of ΔGCA

$= \frac{1}{3} \cdot$ area of ΔABC.

$\therefore$ $ar(\Delta GAB) = \frac{1}{3} \times \left\{ \frac{1}{2} \times 3 \times 4 \right\} = 2$

$\therefore$ Using Circumradius $= R = \frac{abc}{4\Delta}$

Circumradius of $\Delta GAB = \dfrac{AG \times BG \times AB}{4 \times ar(\Delta GAB)}$

$$= \frac{5}{3} \times \frac{2\sqrt{13}}{3} \times 3$$

$$\overline{\quad 4 \times 2 \quad}$$

$$= \frac{5\sqrt{13}}{12} \quad \underline{\underline{\text{Ans.}}}$$

(option C)