Determination of G

1. Newton's Laws of motion :- 2nd & 3rd laws are used here.
2. Concept of Centripital force

$$F_c = mv^2/r$$ (for object moving in circular path)

3. The Galilean Law of freely falling bodies
4. The Laws of Kepler.

$$F_G = +\frac{GMm}{r^2}$$

↳ Gravitational force.

A : M_A : $\vec{R_A}$

B : M_B : $\vec{R_B}$

$$\vec{R_{AB}} = \vec{R_A} - \vec{R_B}$$

$$F_{A \to B} = -\frac{G M_A M_B \vec{R_{AB}}}{R_{AB}^3}$$ (in vector form)

Galilean laws :- acceleration of body is independent both of mass & height.

$$ma = \frac{GmM_E}{(R_E + h)^2}$$

$$\boxed{\frac{h}{R_E} \ll 1}$$

$$a = \frac{GM_E}{h^2}\left[\frac{1}{\left(1+\frac{h}{R_E}\right)^2}\right] \qquad x = \frac{h}{R_E} \ll 1$$

$$\frac{1}{(R_E+h)^2} = \frac{1}{R_E^2\left(1+\left(\frac{h}{R_E}\right)^2\right)}$$

$$= \frac{1}{R_E^2(1+x)^2}$$

$$\frac{1}{(1+x)^2} = \frac{1}{(1+2x+x^2)}$$

$x^2 \ll x \ll 1$

$$\simeq \frac{1}{1+2x} \simeq (1-2x)$$

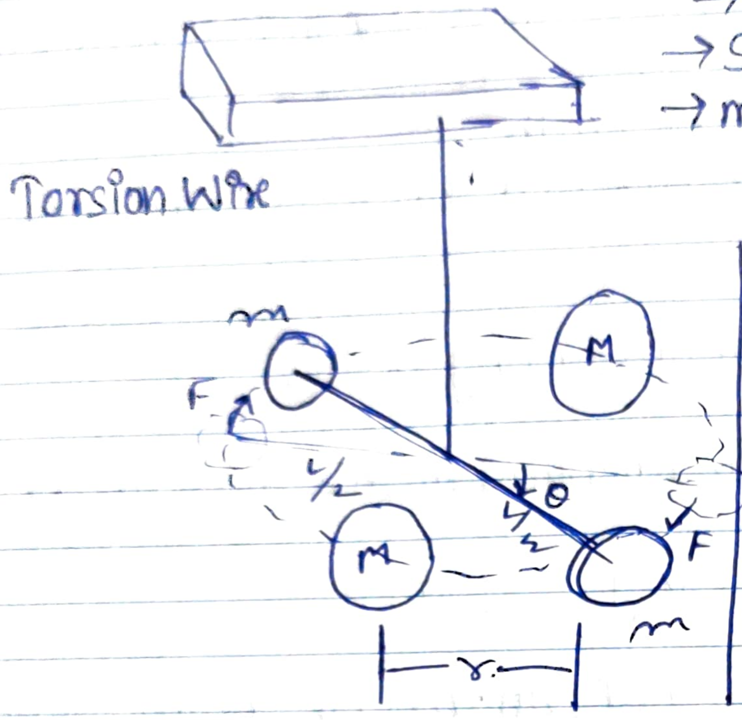
$$ma = \frac{F_G}{m} = \frac{GM_E m}{R_E^2}\left(1-2\frac{h}{R_E}\right)$$

→ Distances are known
→ Masses not known
→ G Value not determined
→ Galilean law determines $M_E G$

$R_E \sim 6400km$; $h \sim 100m$

$$\frac{2h}{R} \sim 3\times10^{-4}$$

* Experimental Details
→ Large lead balls : 158.04 kg
→ Small lead balls : 0.73 kg
→ mass of the rod : 0.03 kg



$\tau = K\theta = LF = L\frac{GMm}{d^2}$

$\therefore G = \frac{K\theta d^2}{LMm}$

: How to determine K?

$T = 2\pi\sqrt{\frac{I}{K}} = 2\pi\sqrt{\frac{mL^2}{2K}}$

$G = \frac{2\pi^2 L d^2}{MT^2}$

G (cavendish) = 6.74×10^{-11}

G (Modern) = 6.67408×10^{-11}

$mg = \frac{GmM_E}{R_E^2}$ (ON EARTH)

$M_E = \frac{gR_E^2}{G} = \frac{4\pi}{3}R_E^3 \rho_E$ (where ρ = density)

$\frac{\rho_E}{\rho_W} = 5.448 \pm 0.033$