Illustration 1: Find the variance of first n natural numbers.

Solution: The variance is given by

$$V = \sigma^2 = \frac{1}{n}\left[\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right]$$

where x_i, i = 1, 2, ..., n are the first n natural numbers.

Now, $\sum x_i = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.

$\sum x_i^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Thus, $V = \frac{1}{n}\left[\frac{1}{6}n(n+1)(2n+1) - \frac{n^2(n+1)^2}{4n}\right]$

$= \frac{n(n+1)}{24n}\left[4(2n+1) - 6(n+1)\right] = \frac{(n+1)}{24}(2n-2) = \frac{n^2-1}{12}$.

Illustration 2: Find the standard deviation of 7 scores 1,2,3,4,5, 6,7.

Solution: Standard deviation of first n natural numbers is $\sqrt{\frac{n^2-1}{12}}$. For n = 7,

the value $= \sqrt{\frac{7^2-1}{12}} = \sqrt{4} = 2$.

Illustration 3: Find the mean and standard deviation for the following data

Age (years)	25–30	30–35	35–40	40–45	45–50	50–55
No. of teachers	30	23	20	14	10	3

Solution: We have a grouped data. The distance between two successive mid-values of the classes is 5, that is h = 5. We choose

a = 42.5 and $u_i = \frac{x_i - a}{h} = \frac{x_i - 42.5}{5}$.

Class	Mid value (x_i)	f_i	u_i	f_iu_i	u_i^2	$f_iu_i^2$
25–30	27.5	30	–3	–90	9	270
30–35	32.5	23	–2	–46	4	92
35–40	37.5	20	–1	–20	1	20
40–45	42.5	14	0	0	0	0
45–50	47.5	10	1	10	1	10
50–55	52.5	3	2	6	4	12
		100		–140		404

$\therefore\ \bar{x} = a + h\left(\frac{\sum f_iu_i}{\sum f_i}\right) = 42.5 + 5\left(-\frac{140}{100}\right) = 35.5$

$$V = \sigma^2 = \frac{h^2}{\sum f_i}\left[\sum f_i u_i^2 - \frac{\left(\sum f_i u_i\right)^2}{\sum f_i}\right] = \frac{25}{100}\left[404 - \frac{(-140)^2}{100}\right] = 52.$$

S.d. = $\sigma = \sqrt{52} \approx 7.21$.

Illustration 4: The scores of 25 students in an intelligence test are given below: 75, 56, 50, 62, 68, 62, 56, 78, 80, 75, 50, 62, 72, 78, 68, 67, 80, 75, 50, 68, 80, 68, 62, 56, 68.
Find the mean and standard deviation of the data.

Solution: We note that the marks are repeated.
Hence, the data foms a frequency distribution. Since the numbers are large varying from 50 to 80, we choose the shift value as a = 65.
We have following frequency distribution.

Salary x_i	Frequency f_i	$d_i = x_i - a$ $= x_i - 65$	$f_i d_i$	d_i^2	$f_i d_i^2$
50	3	−15	−45	225	675
56	3	−9	−27	81	243
62	4	−3	−12	9	36
67	1	2	2	4	4
68	5	3	15	9	45
72	1	7	7	49	49
75	3	10	30	100	300
78	2	13	26	169	338
80	3	15	45	225	675
	25	23	41		2365

Mean = $\bar{x} = a + \bar{d} = 65 + \frac{41}{25} = 66.64$

$$\sigma^2 = \frac{1}{\sum f_i}\left[\sum f_i d_i^2 - \frac{\left(\sum f_i d_i\right)^2}{\sum f_i}\right] = \frac{1}{25}\left[2365 - \frac{(41)^2}{25}\right] = \frac{57444}{625}$$

and S.d. = $\sigma = \frac{\sqrt{57444}}{25} \approx 9.59$.

Illustration 5: Find the variance of 2, 4, 6, 8, 10.

Solution: Variance of 1, 2, 3, 4, 5, is $\frac{n^2 - 1}{12} = \frac{5^2 - 1}{12} = 2$

($\because$ variance of first n natural number is $(n^2 - 1)/12$, when each item is doubled (i.e. 2, 4, 5, 8, 10) variance is multiplied by $2^2 = 4$. Required variance = 4 × 2 = 8)

Illustration 6: The standard deviations of two samples of sizes 50 and 100 are 8 and 7 respectively. Find the standard deviation of the combined sample.

Solution: S.D. of combined sample $= \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2}} = \sqrt{\frac{1}{150}(50 \times 64 + 100 \times 49)} = 7.35$