Chapter 9

Sequences and Series

1. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then : **(2017)**
(a) a, b and c are in G.P. (b) b, c and a are in G.P.
(c) b, c and a are in A.P. (d) a, b and c are in A.P.

2. If three positive numbers a, b and c are in A.P. such that abc = 8, then the minimum possible value of b is : **[Online April 9, 2017]**
(a) 2 (b) $4^{\frac{1}{3}}$
(c) $4^{\frac{2}{3}}$ (d) 4

3. Let $a_1, a_2, a_3, \ldots, a_n$, be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to : **[Online April 10, 2016]**
(a) 306 (b) 204
(c) 153 (d) 612

4. Let α and β be the roots of equation $px^2 + qx + r = 0, p \neq 0$.
If p, q, r are in A.P and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is: **[2014]**
(a) $\frac{\sqrt{34}}{9}$ (b) $\frac{2\sqrt{13}}{9}$
(c) $\frac{\sqrt{61}}{9}$ (d) $\frac{2\sqrt{17}}{9}$

5. The sum of the first 20 terms common between the series 3 + 7 + 11 + 15 + ... and 1 + 6 + 11 + 16 + ..., is **[Online April 11, 2014]**
(a) 4000 (b) 4020
(c) 4200 (d) 4220

6. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4^{th} term is: **[Online April 9, 2014]**
(a) 8 (b) 16
(c) 20 (d) 24

7. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P., is : **[Online April 23, 2013]**
(a) 10m (b) 12m
(c) 13m (d) 15m

8. Given sum of the first n terms of an A.P. is $2n + 3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is : **[Online April 22, 2013]**
(a) $n + 4n^2$ (b) $6n^2 - n$
(c) $n^2 + 4n$ (d) $3n + 2n^2$

9. Let $a_1, a_2, a_3, \ldots$ be an A.P, such that
$\frac{a_1 + a_2 + \ldots + a_p}{a_1 + a_2 + a_3 + \ldots + a_q} = \frac{p^3}{q^3}; p \neq q$. Then $\frac{a_6}{a_{21}}$ is equal to: **[Online April 9, 2013]**
(a) $\frac{41}{11}$ (b) $\frac{31}{121}$
(c) $\frac{11}{41}$ (d) $\frac{121}{1861}$

10. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is : **[2012]**
(a) -150 (b) 150 times its 50^{th} term
(c) 150 (d) Zero

11. If the A.M. between p^{th} and q^{th} terms of an A.P. is equal to the A.M. between r^{th} and s^{th} terms of the same A.P., then $p + q$ is equal to **[Online May 26, 2012]**
(a) $r + s - 1$ (b) $r + s - 2$
(c) $r + s + 1$ (d) $r + s$

12. Suppose θ and $\phi\ (\neq 0)$ are such that $\sec(\theta + \phi)$, $\sec\theta$ and $\sec(\theta - \phi)$ are in A.P. If $\cos\theta = k\cos\left(\frac{\phi}{2}\right)$ for some k, then k is equal to **[Online May 19, 2012]**
(a) $\pm\sqrt{2}$ (b) ± 1
(c) $\pm\frac{1}{\sqrt{2}}$ (d) ± 2

Hints & Solutions

1. **(c)** We have

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$\Rightarrow 225a^2 + 9b^2 + 25c^2 - 75ac = 45ab + 15bc$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 75ac - 45ab - 15bc = 0$

$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

it is possible when $15a - 3b = 0$, $3b - 5c = 0$ and $5c - 15a = 0$

$\Rightarrow 15a = 3b = 5$

$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$

$\Rightarrow a + b = \frac{c}{3} + \frac{5c}{3} = \frac{6c}{3}$

$\Rightarrow a + b = 2c$

$\Rightarrow$ b, c, a are in A.P.

2. **(a)** **By Arithmetic Mean**:

$a + c = 2b$

Consider $a = b = c = 2$

$\Rightarrow$ $abc = 8$

$\Rightarrow$ $a + b = 2b$

$\therefore$ minimum possible value of $b = 2$

3. **(a)** $a_3 + a_7 + a_{11} + a_{15} = 72$

$(a_3 + a_{15}) + (a_7 + a_{11}) = 72$

$a_3 + a_{15} + a_7 + a_{11} = 2(a_1 + a_{17})$

$a_1 + a_{17} = 36$

$S_{17} = \frac{17}{2}[a_1 + a_{17}] = 17 \times 18 = 306$

4. **(b)** Let p, q, r are in AP

$\Rightarrow 2q = p + r$ ...(i)

Given $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = 4$

We have $\alpha + \beta = -q/p$ and $\alpha\beta = \frac{r}{p}$

$\Rightarrow \frac{-\frac{q}{p}}{\frac{r}{p}} = 4 \Rightarrow q = -4r$ ....(ii)

From (i), we have

$2(-4r) = p + r$

$p = -9r$

$q = -4r$

$r = r$

Now $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}} = \frac{\sqrt{q^2 - 4pr}}{|p|}$

$= \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$

5. **(b)** Given n = 20; S_{20} = ?

Series (1) $\rightarrow$ 3, 7, <u>11</u>, 15, 19, 23, 27, <u>31</u>, 35, 39, 43, 47, <u>51</u>, 55, 59...

Series (2) $\rightarrow$ 1, 6, <u>11</u>, 16, 21, 26, <u>31</u>, 36, 41, 46, <u>51</u>, 56, 61, 66, 71.

The common terms between both the series are 11, 31, 51, 71...

Above series forms an Arithmetic progression (A.P).

Therefore, first term (a) = 11 and

common difference (d) = 20

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2 \times 11 + (20 - 1)\, 20]$

$S_{20} = 10\,[22 + 19 \times 20]$

$S_{20} = 10 \times 402 = 4020$

$\therefore S_{20} = 4020$

6. **(c)** Let a be the first term and d be the common difference of given A.P.

Second term, $a + d = 12$ ...(1)

Sum of first nine terms,

$S_9 = \frac{9}{2}(2a + 8d) = 9(a + 4d)$

Given that S_9 is more than 200 and less than 220

$\Rightarrow 200 < S_9 < 220$

$\Rightarrow 200 < 9(a + 4d) < 220$

$\Rightarrow 200 < 9(a + d + 3d) < 220$

Putting value of $(a + d)$ from equation (1)

$200 < 9(12 + 3d) < 220$

$\Rightarrow 200 < 108 + 27d < 220$

$\Rightarrow 200 - 108 < 108 + 27d - 108 < 220 - 108$

$\Rightarrow 92 < 27d < 112$

Possible value of d is 4

$27 \times 4 = 108$

Thus, 92 < 108 < 112
Putting value of d in equation (1)
$a + d = 12$
$a = 12 - 4 = 8$
4^{th} term $= a + 3d = 8 + 3 \times 4 = 20$

7. **(c)** If d be the common difference, then

$m = a_4 - a_7 + a_{10} = a_4 - a_7 + a_7 + 3d = a_7$

$$S_{13} = \frac{13}{2}[a_1 + a_{13}] = \frac{13}{2}[a_1 + a_7 + 6d]$$

$$= \frac{13}{2}[2a_7] = 13a_7 = 13\text{ m}$$

8. **(b)** Given $S_n = 2n + 3n^2$
Now, first term $= 2 + 3 = 5$
second term $= 2(2) + 3(4) = 16$
third term $= 2(3) + 3(9) = 33$
Now, sum given in option (b) only has the same first term and difference between 2nd and 1st term is double also.

9. **(b)** $$\frac{a_1 + a_2 + a_3 + ...+ a_p}{a_1 + a_2 + a_3 + ...+ a_q} = \frac{p^3}{q^3}$$

$$\Rightarrow \frac{a_1 + a_2}{a_1} = \frac{8}{1} \Rightarrow a_1 + (a_1 + d) = 8a_1$$

$\Rightarrow d = 6a_1$

Now $$\frac{a_6}{a_{21}} = \frac{a_1 + 5d}{a_1 + 20d}$$

$$= \frac{a_1 + 5 \times 6a_1}{a_1 + 20 \times 6a_1} = \frac{1+30}{1+120} = \frac{31}{121}$$

10. **(d)** Let 100^{th} term of an AP is $a + (100 - 1)\, d = a + 99d$ where 'a' is the first term of $A.P$ and 'd' is the common difference of $A.P$.
Similarly, 50^{th} term $= a + (50 - 1)\, d$
$= a + 49d$
Now, According to the question
$100\,(a + 99d) = 50\,(a + 49d)$
$\Rightarrow 2a + 198\,d = a + 49d \Rightarrow a + 149\,d = 0$
This is the 150^{th} term of an $A.P$.
Hence, $T_{150} = a + 149\,d = 0$

11. **(d)** Given : $$\frac{a_p + a_q}{2} = \frac{a_r + a_s}{2}$$

$\Rightarrow a + (p - 1)\,d + a + (q - 1)d$
$= a + (r - 1)d + a + (s - 1)d$
$\Rightarrow 2a + (p + q)d - 2d = 2a + (r + s)\,d - 2d$
$\Rightarrow (p + q)d = (r + s)d \Rightarrow p + q = r + s.$

12. **(a)** Since, $\sec(\theta - \phi)$, $\sec\theta$ and $\sec(\theta + \phi)$ are in A.P.,
$\therefore 2\sec\theta = \sec(\theta - \phi) + \sec(\theta + \phi)$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{\cos(\theta+\phi) + \cos(\theta-\phi)}{\cos(\theta-\phi)\cos(\theta+\phi)}$$

$$\Rightarrow 2\left(\cos^2\theta - \sin^2\phi\right) = \cos\theta[2\cos\theta\cos\phi]$$

$$\Rightarrow \cos^2\theta(1 - \cos\phi) = \sin^2\phi = 1 - \cos^2\phi$$

$$\Rightarrow \cos^2\theta = 1 + \cos\phi = 2\cos^2\frac{\phi}{2}$$

$$\therefore \cos\theta = \sqrt{2}\cos\frac{\phi}{2}$$

But given $\cos\theta = k\cos\frac{\phi}{2}$

$\therefore k = \sqrt{2}$