Notes and formulas – Lecture 4

Viscosity

The property of a fluid by virtue of which an internal frictional force acts between its different layers which opposes their relative motion is called **viscosity.**

These internal frictional force is called **viscous force.**

Viscous forces are intermolecular forces acting between the molecules of different layers of liquid moving with different velocities.

$$\text{Viscous force } (F) = -\eta\, A \frac{dv}{dx}$$

or

$$\eta = -\frac{F}{A\left(\frac{dv}{dx}\right)}$$

where, $\frac{dv}{dx}$ = rate of change of velocity with distance called velocity gradient, A = area of cross-section and η = coefficient of viscosity.

SI unit of η is Nsm^{-2} or pascal-second or decapoise. Its dimensional formula is $[ML^{-1}T^{-1}]$.

The knowledge of the coefficient of viscosity of different oils and its variation with temperature helps us to select a suitable lubricant for a given machine.

The cause of viscosity in liquid is due to cohesive force between liquid molecules, while in gases, it is due to diffusion.

Viscosity is due to transport of momentum. The value of viscosity (and compressibility) for ideal liquid is zero.

The viscosity of air and of some liquids is utilised for damping the moving parts of some instruments.

The knowledge of viscosity of some organic liquids is used in determining the molecular weight and shape of large organic moleculars like proteins and cellulose.

In any layer of liquid, the pulling of lower layers backwards while upper layers forward direction is known as laminar flow.

Poiseuille's Formula

The rate of flow (v) of liquid through a horizontal pipe for steady flow is given by

$$v = \frac{\pi}{8} \frac{pr^4}{\eta l}$$

where, p = pressure difference across the two ends of the tube, r = radius of the tube, η = coefficient of viscosity and l = length of the tube.

Rate of Flow of Liquid

Rate of flow of liquid through a tube is given by

$$v = \frac{p}{R}$$

where, $R = \frac{8\eta l}{\pi r^4}$ called liquid resistance and p = liquid pressure.

Surface tension is the property of any liquid by virtue of which it tries to minimise its free surface area.

Surface tension of a liquid is measured as the force acting per unit length on an imaginary line drawn tangentially on the free surface of the liquid.

Surface tension, $S = \frac{\text{Force}}{\text{Length}} = \frac{F}{l} = \frac{\text{Work done}}{\text{Change in area}}$

Its SI unit is Nm^{-1} or Jm^{-2} and its dimensional formula is $[MT^{-2}]$.

It is a scalar quantity. Surface tension is a molecular phenomenon which is due to cohesive force.

Surface tension of a liquid depends only on the nature of liquid and is independent of the surface area of film or length of the line considered.

Small liquid drops are spherical due to the property of surface tension.

Surface Energy

If we increase the free surface area of a liquid, then work has to be done against the force of surface tension. This work done is stored in liquid surface as potential energy.

This additional potential energy per unit area of free surface of liquid is called surface energy.

$$\text{Surface energy } (E) = S \times \Delta A$$

where, S = surface tension and ΔA = increase in surface area.

(i) **Work Done in Blowing a Liquid Drop** If a liquid drop is blown up from a radius r_1 to r_2, then work done for that is

$$W = S \cdot 4\pi(r_2^2 - r_1^2)$$

(ii) **Work Done in Blowing a Soap Bubble** As a soap bubble has two free surfaces, hence work done in blowing a soap bubble so as to increase its radius from r_1 to r_2 is given by

$$W = S \cdot 8\pi(r_2^2 - r_1^2)$$

(iii) **Work Done in Splitting a Bigger Drop into *n* Smaller Droplets**

If a liquid drop of radius R is split up into n smaller droplets, all of same size, then radius of each droplet

$$r = R \cdot (n)^{-1/3}$$

$$\text{Work done, } W = 4\pi S(nr^2 - R^2) = 4\pi SR^2 (n^{1/3} - 1)$$

(iv) **Coalescance of Drops** If n small liquid drops of radius r each combine together so as to form a single bigger drop of radius $R = n^{1/3} \cdot r$, then in the process energy is released. Release of energy is given by $\Delta U = S \cdot 4\pi(nr^2 - R^2) = 4\pi Sr^2 n(1 - n^{-1/3})$

Capillarity

The phenomenon of rise or fall of liquid column in a capillary tube is called capillarity.

Ascent of a liquid column in a capillary tube is given by

$$h = \frac{2S\cos\theta}{r\rho g} - \frac{r}{3}$$

If capillary is very narrow, then

$$h = \frac{2S\cos\theta}{r\rho g}$$

where, r = radius of capillary tube, ρ = density of the liquid,

θ = angle of contact and S = surface tension of liquid.

- If $\theta < 90^\circ$, $\cos\theta$ is positive, so h is positive, *i.e.* liquid rises in a capillary tube.
- If $\theta > 90^\circ$, $\cos\theta$ is negative, so h is negative, *i.e.* liquid falls in a capillary tube.
- Rise of liquid in a capillary tube does not violate law of conservation of energy.