

Example 20 The number of terms in the expansion of $(a + b + c)^n$, where $n \in \mathbf{N}$ is

(A) $\frac{(n+1)(n+2)}{2}$

(B) $n + 1$

(C) $n + 2$

(D) $(n + 1)n$

Solution A is the correct choice. We have

$$\begin{aligned}(a + b + c)^n &= [a + (b + c)]^n \\ &= a^n + {}^n C_1 a^{n-1} (b + c)^1 + {}^n C_2 a^{n-2} (b + c)^2 \\ &\quad + \dots + {}^n C_n (b + c)^n\end{aligned}$$

Further, expanding each term of R.H.S., we note that

First term consist of 1 term.

Second term on simplification gives 2 terms.

Third term on expansion gives 3 terms.

Similarly, fourth term on expansion gives 4 terms and so on.

$$\text{The total number of terms} = 1 + 2 + 3 + \dots + (n + 1)$$

$$= \frac{(n+1)(n+2)}{2}$$