**Example 15** If $a_1$, $a_2$, $a_3$ and $a_4$ are the coefficient of any four consecutive terms in the expansion of $(1 + x)^n$, prove that

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$$

**Solution** Let $a_1$, $a_2$, $a_3$ and $a_4$ be the coefficient of four consecutive terms $T_{r+1}$, $T_{r+2}$, $T_{r+3}$, and $T_{r+4}$ respectively. Then

$$a_1 = \text{coefficient of } T_{r+1} = {}^nC_r$$

$$a_2 = \text{coefficient of } T_{r+2} = {}^nC_{r+1}$$

$$a_3 = \text{coefficient of } T_{r+3} = {}^nC_{r+2}$$

and $$a_4 = \text{coefficient of } T_{r+4} = {}^nC_{r+3}$$

Thus $$\frac{a_1}{a_1 + a_2} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}}$$

$$= \frac{{}^nC_r}{{}^{n+1}C_{r+1}} \qquad (\because \quad {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$$

$$= \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} \times \frac{\underline{|r+1}\,\underline{|n-r}}{\underline{|n+1}} = \frac{r+1}{n+1}$$

Similarly, $$\frac{a_3}{a_3 + a_4} = \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}}$$

$$= \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} = \frac{r+3}{n+1}$$

Hence, $$\text{L.H.S.} = \frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2r+4}{n+1}$$

and $$\text{R.H.S.} = \frac{2a_2}{a_2 + a_3} = \frac{2\left({}^nC_{r+1}\right)}{{}^nC_{r+1} + {}^nC_{r+2}} = \frac{2\left({}^nC_{r+1}\right)}{{}^{n+1}C_{r+2}}$$

$$= 2\frac{\underline{|n}}{\underline{|r+1}\,\underline{|n-r-1}} \times \frac{\underline{|r+2}\,\underline{|n-r-1}}{\underline{|n+1}} = \frac{2(r+2)}{n+1}$$