Example 11 Find numerically the greatest term in the expansion of $(2 + 3x)^9$, where $x = \frac{3}{2}$.

Solution We have $(2 + 3x)^9 = 2^9\left(1+\frac{3x}{2}\right)^9$

Now,
$$\frac{T_{r+1}}{T_r} = \frac{2^9\left[{}^9C_r\left(\frac{3x}{2}\right)^r\right]}{2^9\left[{}^9C_{r-1}\left(\frac{3x}{2}\right)^{r-1}\right]}$$

$$= \frac{{}^9C_r}{{}^9C_{r-1}}\left|\frac{3x}{2}\right| = \frac{\underline{|9}}{\underline{|r}\,\underline{|9-r}}\cdot\frac{\underline{|r-1}\,\underline{|10-r}}{\underline{|9}}\left|\frac{3x}{2}\right|$$

$$= \frac{10-r}{r}\left|\frac{3x}{2}\right| = \frac{10-r}{r}\left(\frac{9}{4}\right) \qquad \text{Since } x = \frac{3}{2}$$

Therefore,
$$\frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{90-9r}{4r} \geq 1$$

$$\Rightarrow 90 - 9r \geq 4r \qquad \text{(Why?)}$$

$$\Rightarrow r \leq \frac{90}{13}$$

$$\Rightarrow r \leq 6\frac{12}{13}$$

Thus the maximum value of r is 6. Therefore, the greatest term is $T_{r+1} = T_7$.

Hence,
$$T_7 = 2^9\left[{}^9C_6\left(\frac{3x}{2}\right)^6\right], \qquad \text{where } x = \frac{3}{2}$$

$$= 2^9 \cdot {}^9C_6\left(\frac{9}{4}\right)^6 = 2^9 \cdot \frac{9\times8\times7}{3\times2\times1}\left(\frac{3^{12}}{2^{12}}\right) = \frac{7\times3^{13}}{2}$$