Example 21 The ratio of the coefficient of x^{15} to the term independent of x in

$\left(x^2 + \dfrac{2}{x}\right)^{15}$ is

(A) 12:32   (B) 1:32   (C) 32:12   (D) 32:1

Solution (B) is the correct choice. Let T_{r+1} be the general term of $\left(x^2 + \dfrac{2}{x}\right)^{15}$, so,

$$T_{r+1} = {}^{15}C_r \, (x^2)^{15-r} \left(\frac{2}{x}\right)^r$$

$$= {}^{15}C_r \, (2)^r \, x^{30-3r} \qquad\qquad\qquad \dots (1)$$

Now, for the coefficient of term containing x^{15},

$$30 - 3r = 15, \quad \text{i.e.,} \quad r = 5$$

Therefore, ${}^{15}C_5 \, (2)^5$ is the coefficient of x^{15} (from (1))

To find the term independent of x, put $30 - 3r = 0$

Thus ${}^{15}C_{10} \, 2^{10}$ is the term independent of x (from (1))

Now the ratio is $\dfrac{{}^{15}C_5 \, 2^5}{{}^{15}C_{10} \, 2^{10}} = \dfrac{1}{2^5} = \dfrac{1}{32}$