

**Example 18** If  $(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \dots + a_{2n} x^{2n}$ , then  $a_0 + a_2 + a_4 + \dots + a_{2n}$  equals.

- (A)  $\frac{3^n + 1}{2}$       (B)  $\frac{3^n - 1}{2}$       (C)  $\frac{1 - 3^n}{2}$       (D)  $3^n + \frac{1}{2}$

**Solution** A is the correct choice. Putting  $x = 1$  and  $-1$  in

$$(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \dots + a_{2n} x^{2n}$$

we get  $1 = a_0 + a_1 + a_2 + a_3 + \dots + a_{2n}$  ... (1)

and  $3^n = a_0 - a_1 + a_2 - a_3 + \dots + a_{2n}$  ... (2)

Adding (1) and (2), we get

$$3^n + 1 = 2(a_0 + a_2 + a_4 + \dots + a_{2n})$$

Therefore  $a_0 + a_2 + a_4 + \dots + a_{2n} = \frac{3^n + 1}{2}$