Example 17 If the coefficients of x^7 and x^8 in $\left(2+\dfrac{x}{3}\right)^n$ are equal, then n is

(a) 56 (b) 55 (c) 45 (d) 15

Solution B is the correct choice. Since $T_{r+1} = {}^nC_r\, a^{n-r} x^r$ in expansion of $(a + x)^n$,

Therefore, $$T_8 = {}^nC_7 (2)^{n-7} \left(\frac{x}{3}\right)^7 = {}^nC_7 \frac{2^{n-7}}{3^7} x^7$$

and $$T_9 = {}^nC_8\, (2)^{n-8} \left(\frac{x}{3}\right)^8 = {}^nC_8 \frac{2^{n-8}}{3^8} x^8$$

Therefore, $${}^nC_7 \frac{2^{n-7}}{3^7} = {}^nC_8 \frac{2^{n-8}}{3^8}$$ (since it is given that coefficient of x^7 = coefficient x^8)

$$\Rightarrow \quad \frac{\lfloor\underline{n}}{\lfloor\underline{7}\,\lfloor\underline{n-7}} \times \frac{\lfloor\underline{8}\,\lfloor\underline{n-8}}{\lfloor\underline{n}} = \frac{2^{n-8}}{3^8} \cdot \frac{3^7}{2^{n-7}}$$

$$\Rightarrow \quad \frac{8}{n-7} = \frac{1}{6} \quad \Rightarrow \quad n = 55$$